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PRINCIPLES OF GUNNERY.

RIFLED ORDNANCE.

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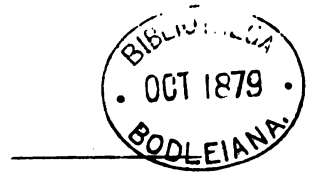
RIFLED ORDNANCE.

BY

MAJOR J. SLADEN, R.A.

(Professor of Artillery, R.M. Academy, Woolwich).



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PREFACE.

It has been the object of the author, in the following pages, to elucidate the principles involved in gunnery science, so far as they bear upon questions in practice. It is hoped that it will be found a useful addendum to the Treatises on Ordnance, Ammunition, and Carriages published by the War Office. The author gratefully acknowledges the assistance given him by Professor Bashforth, Mr. W. D. Niven, of Trinity College, Cambridge, as well as the courtesy of Mr. Greenhill, Professor of Mathematics to the Advanced Class of Artillery Officers, in revising the proof sheets.

R.M. Academy,
May 20th, 1879.

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Object of rifling guns.

With smooth-bore guns, a frequent cause of inaccuracy at moderate ranges is due to the uncertain rotation communicated to the projectile in the bore of the gun; so that it never can be confidently predicted whether the projectile will deflect to the right or left. With rifled guns, on the contrary, rotation is impressed on the projectile by means of spiral grooves cut in the gun; so that the rotation is no longer uncertain, but always takes place in a fixed direction, giving a constant deflection, which can be allowed for in laying the gun.

Moreover, it becomes possible, by giving sufficient rotation to an elongated projectile, to keep it point first during its flight through the

air; and so to diminish the effects of the resistance of the air,* by increasing the ratio of the weight of the projectile to the diameter of the bore, and also by giving the head of the projectile any desired shape. Hence arises greater accuracy, greater effect of shell, greater penetration, and a flatter trajectory.

The intensity of effect is much greater, then, with rifled than with smooth-bore ordnance, and any required result can be attained with a much less expenditure of time and *matériel.*

Accuracy of fire, combined with *effect* of shell, are the great objects to be attained by rifled ordnance.

Accuracy of fire increases (*cæteris paribus*) with the velocity. The effect of shell increases with its weight, and its capacity for holding a larger bursting charge or a greater number of bullets.

Projectile centred. In order to secure the greatest accuracy of fire, it is necessary that the axis of the projectile should coincide at the muzzle with the axis of the piece from which it is fired. The projectile is then said to be *centred.*

Axis of the piece. † The *axis of the piece* is an imaginary line passing down the centre of the bore.

Axis of the trunnions. The *axis of the trunnions* is an imaginary line passing through the centre of the trunnions at right angles to the axis of the piece.

Line of sight. The *line of sight* is an imaginary line passing through the sights of the piece and the point aimed at. For practical purposes this may be termed the *line of fire.*

Plane of sight. The *plane of sight* is the vertical plane passing through the line of sight.

Angle of sight. The *angle of sight* is the angle which the line of sight makes with the horizontal plane.

Angle of elevation. The *angle of elevation* is the angle which the line of sight makes with the axis of the piece.

Quadrant angle. The *quadrant angle* is the angle which the axis of the piece, when laid, makes with the horizontal plane, and is termed *quadrant elevation* or *depression,* according as the piece is laid above or below the horizontal plane.

Trajectory. The *trajectory* is the curve described by the projectile in passing from the muzzle of the piece to the first point of impact.

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* *Vide* pamphlet by the author, "The Merits of a Large Bore and Small Bore Contrasted." ("Proceedings, R.A. Institution," Vol. VII. p. 273.

† *Vide* Report of Committee on Artillery Terms, 16th February, 1878.

The *line of departure* is the direction in which the projectile is moving on leaving the piece; or, in other words, a tangent to the trajectory at the muzzle. Line of departure.

The *plane of departure* is the vertical plane passing through the line of departure and the axis of the piece, when laid. Plane of departure.

The *angle of departure* is the angle between the line of departure and the horizontal plane. The excess of the angle of departure above the quadrant angle is commonly called the *jump*. Angle of departure.

The *angle of projection* is the angle between the line of departure and the line of sight. Angle of projection.

The *angle of descent* is the angle which a tangent to the trajectory at the first point of impact makes with the horizontal plane. Angle of descent.

The *range* is the distance from the muzzle of the piece to the intersection of the trajectory with the line of sight. Range.

The *lateral deviation* is the perpendicular distance of the point of impact of the projectile right or left of the plane of sight. Lateral deviation.

The *drift* is the constant deflection of a projectile from the plane of departure due to the rotation imparted by the rifling of the piece. It is sometimes termed *derivation*. Drift.

The terms distinguishing the various natures of artillery fire have reference (1) to the conditions within the *vertical plane;* (2) to the conditions within the *horizontal plane*. * Terms distinguishing the various natures of artillery fire.

(1) The terms denoting the conditions within the vertical plane are *direct* fire, *indirect* or *curved* fire, and *high-angle* fire.

Direct fire is the fire from guns with service charges at all angles of elevation not exceeding 15°. Direct fire.

Indirect or *curved* fire is the fire from guns with reduced charges, and from howitzers and mortars at all angles of elevation not exceeding 15°. Indirect or curved fire.

High-angle fire is the fire from guns, howitzers, and mortars at all angles of elevation exceeding 15°. High-angle fire.

(2) The terms denoting the conditions within the horizontal plane are *front, oblique, enfilade,* and *reverse* fire. These terms are best explained by means of a diagram.

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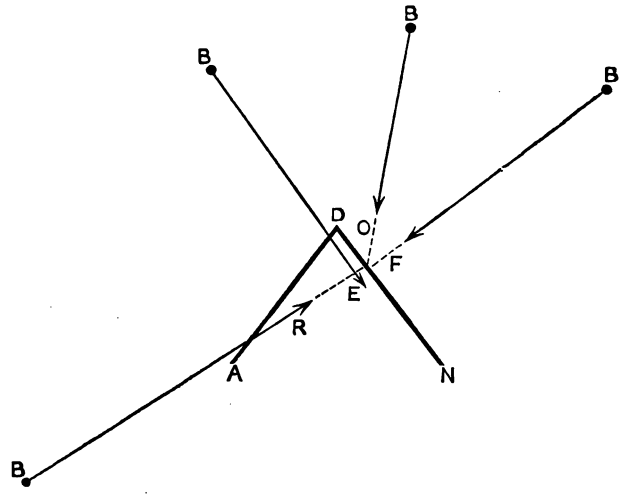
* *Vide* Regimental Order dated D.-A.-G.'s Office, 31st January, 1877.



ADN represents the plan of work fired at, *B* the position of attacking batteries. Then *BF*, perpendicular to the face, *DN*, of the work, represents the general direction of *front* fire; *BO*, oblique to *DN*, represents the general direction of *oblique* fire; *BE*, parallel to the face *DN*, represents the general direction of *enfilade* fire; and *BR*, perpendicular to the reverse slope of *DN*, represents the general direction of *reverse* fire.

Front fire. Oblique fire. Enfilade fire. Reverse fire.

A combination of these terms allows of an accurate description of the precise nature of artillery fire employed; *e.g.*, *direct front* fire denotes that the fire is *direct* in the vertical plane, and *front* in the horizontal plane.

Muzzle velocity.

The *muzzle velocity* of a projectile is its velocity on leaving the piece.

Muzzle velocity is generally measured in feet per second (abbreviated thus—"f.s.")

Resistance of the air.

The *resistance of the air* to a projectile in motion, is the force which tends to stop the projectile, and so continually diminishes its velocity. It is generally measured in lbs. avoirdupois.

Remaining velocity.

The *remaining velocity* of a projectile is its velocity at any given point of its trajectory. The remaining velocity of a projectile must in all cases be less than the muzzle velocity, owing to the resistance of the air.

Retardation.

Retardation is the *rate* at which the projectile loses velocity, owing to the resistance of the air.

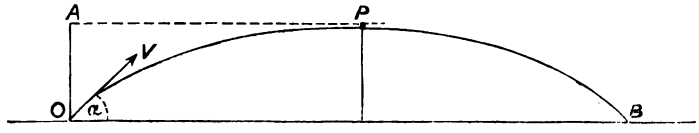
The *striking velocity* of a projectile is the velocity which it has on impact. Striking velocity.

The *horizontal velocity* of a projectile is its velocity at any point resolved in a horizontal direction. Horizontal velocity.

The *vertical velocity* of a projectile is its velocity at any point resolved in a vertical direction. Thus, if V be the velocity of a projectile moving at an angle α to the horizon, Vertical velocity.

$$\text{Horizontal velocity} = V\cos\alpha,$$
$$\text{Vertical} \quad , \quad = V\sin\alpha.$$

Gravity is that force (caused by the attraction of the earth) which continually decreases the vertical velocity of the projectile in its ascending path, and then brings the projectile to *rest* in a *vertical* direction. Gravity then causes it to descend, and continually increases its descending vertical velocity till it strikes the ground. Gravity.



Thus, if OV be the line of fire, α the angle of fire, and V the muzzle velocity of a projectile, its ascending vertical velocity will be

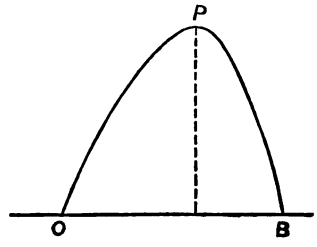
$$V\sin\alpha;$$

or if $V = 650$ f.s., and $\alpha = 30$,

$$\text{ascending vertical velocity} = 650 \times \tfrac{1}{2} = 325 \text{ f.s.}$$

Gravity, and retardation due to the resistance of the air, combine to bring the projectile to rest at some point, A, if it were projected vertically upwards; but owing to the horizontal velocity of the projectile, it will have moved on at the same time to the point P, which is called the vertex, or *maximum height*, of the trajectory. Then the projectile commences to fall, and acquires a *descending vertical velocity* (under the influence of gravity *less* the retardation due to the resistance of the air), which *gradually increases* till the projectile strikes the ground. The horizontal velocity meanwhile has carried the projectile to the point B.

But when a projectile is fired at a high angle of elevation with considerable muzzle velocity, and consequently has a long vertical



distance to fall through in the descending part of the trajectory, there becomes, owing to the resistance of the air (which increases very rapidly as the projectile is acquiring velocity under the action of gravity), a *limit* to the descending vertical velocity which it acquires; so that the descending vertical velocity ceases to increase, and becomes practically uniform.

This limiting vertical velocity is acquired when the *resistance of the air* to the descending projectile in a vertical direction has become equal to the *weight of the projectile;* and it is called the *terminal velocity* —*i.e.*, the maximum descending vertical velocity which a projectile of given weight could acquire on being let fall from any great height.

Terminal velocity.

The heavier the projectile, the greater the height it must fall through to attain to its terminal velocity.

Calibre. The *calibre* of a rifled gun is the diameter of the bore generally measured across the *lands*.

Windage. In the Armstrong breech-loading system the projectile is made to fit tightly into the grooves, ensuring perfect *centring;* but in all muzzle-loading systems there must be sufficient play between the dimensions of the projectile and those of the bore of the gun to allow of easy loading. This play—or *windage* as it is called—is the difference between the sectional area of the gun through its grooves and that of the body of the projectile through its studs: and is measured practically by the actual linear difference between the calibre of the gun and the diameter of the body of the projectile,* which is called *windage over body;*† also by the linear difference between the diameter through the grooves of the gun and the diameter through the studs of the projectile, which is called *windage over studs;* and also by the linear difference between the width of the groove in the gun and the width of the stud on the projectile, which is called the *play of the stud.*

Clearance. *Clearance* is the linear distance between the body of the projectile and the bore of the gun. The amount of clearance before the projectile moves is the difference between the depth of the groove in the gun and the projection of the stud on the projectile; but when the projectile is in motion and centred in the bore, the clearance all round the projectile will be half the windage over body.

It is very important to reduce the windage as much as possible consistently with ease and rapidity of loading, in order to obtain accuracy of fire; but, on the other hand, it is most important that the projectile should not *jam* in loading. Sufficient windage‡ should therefore be given to render a jam improbable under ordinary conditions of service.

All projectiles are gauged before issue into the service, by passing over them hollow cylinder gauges smaller than the calibre of the gun; and all guns are gauged by a solid, cylindrical, studded plug, larger than the projectile, so as to ensure sufficient limits for easy loading. By this system of gauging, all manufacturing errors liable to cause a jam are detected and rectified.

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* *Vide* "Treatise on Ammunition," p. 140.

† With projectiles having a copper cup attached to their base, to give the necessary rotation *without studs*, the windage is simply measured by the linear difference between the calibre of the gun and the diameter of the projectile.

‡ *Vide* "Treatise on Ammunition," p. 210.

But, on service, studs on the projectile are liable to be set up and to corrode; also the gun becomes foul from continued firing; so that a certain fixed amount of windage is absolutely necessary, both over the body and over the studs. The dimensions of the hollow cylinder gauge and the solid plug gauge are adjusted accordingly, so as to ensure the requisite amount of windage being given which has been found practically necessary for the ordinary conditions of service.

There are certain disadvantages in having windage which it has been the object of artillerists to nullify, or mitigate as much as possible. They are:— Disadvantages of windage.

(1) Loss of powder gas, which escapes by the windage without increasing the velocity of the projectile.

(2) The scoring, or guttering effect which this rush of gas has on the bore of the gun, after a few rounds are fired. In the larger nature of guns, this scoring becomes of serious importance: the upper portion of the bore—where the rush is greatest—becoming excessively scored, rendering the gun unserviceable.

(3) The compression of the studs upon which the weight of the projectile rests when rammed home, owing to the great pressure caused by the rush of gas over the projectile at the first instant of discharge. In some cases, the lower studs—*i.e.*, the studs which bear the weight of the projectile—become so compressed that the clearance is abolished, and the body of the projectile comes into contact with the grooves of the gun; thus tending to strain inordinately both the projectile and the gun. When this has been the case, an examination of the recovered projectile usually shows the marks of the groove of the gun on its body.

(4) The liability to non-centring, and so diminishing the range and accuracy of the gun.

These disadvantages have to a great extent been obviated in the larger nature of guns by the employment of a copper gas-check at the base of the projectile, which is expanded into the grooves of the gun, &c., by the shock of discharge; so that the projectile is centred and the windage sealed.

Windage is useful in the M.L. guns, as it allows the rush of heated gas over the projectile at the moment of explosion to ignite the quick-match in the time fuze, and to set it in action; but with reduced charges, such as are used in *indirect* or *curved fire*, the heat developed is not sufficient to ensure the ignition of the quickmatch. The fuze in such cases is wrapped round with a strand of gun-cotton, to make ignition certain.*

On the other hand, with percussion fuzes having safety-pins (such as the Royal Laboratory percussion fuze), which are withdrawn when the fuze is fixed in the shell, some arrangement must be made to prevent the heated gas penetrating through the hole thus made, and so causing premature explosion.

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* *Vide* "Treatise on Ammunition," p. 18.

CHAPTER II.

TERMS USED IN GUNNERY.

(Continued.)

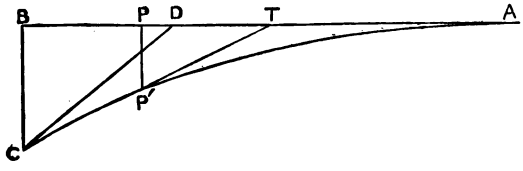
Rifling.—Angle of Spiral.—Twist of Rifling.—Relation between Angle of Spiral and Twist of Rifling.—Determination of the Twist of Rifling at any point in the Bore.—Determination of Linear Rotation of the Exterior of the Projectile, while passing down the Bore.—Examination of recovered Projectiles for groove-marks on Body and Studs.—The Motion of Translation and Rotation of a Projectile fired from Rifled Ordnance.—Velocity of Translation.—Velocity of Rotation.—Linear Velocity of Rotation.—Work.—Pressure per square inch on the Bore of the Gun.—Work Done per square inch.—Energy.—Energy per inch of Circumference.—Energy due to Translation.—Energy due to Rotation.—Total Energy.—Pressure required to give Rotation to Projectiles.—Velocity of Recoil.—Gun Pendulum.—Experiments on Recoil.—Energy of Recoil.—How the Energy of Recoil may be Reduced.—Kemmis's Table of Service M.L.R. Ordnance.

Rifling. All rifled guns have grooves cut spirally in the bore, in order to give the projectile rotation round its longer axis.

Angle of spiral. The angle of spiral (or angle of rifling) differs in different guns, depending principally on the length, weight, and muzzle velocities of the projectiles fired from them.

It is measured at any point of the groove, in guns rifled with *increasing twist* by the angle which a tangent to the groove at that point makes with a line in the bore parallel to its axis; and in guns rifled with *uniform twist*, by the angle which the groove itself makes with a line in the bore parallel to its axis.

Suppose AB to represent the length of the rifled part of the bore of a gun rifled with increasing twist, AC the groove with increasing twist, AB the direction of the axis of the bore—A at the breech end, B at the muzzle.



Then the angle of spiral at the point P' of the groove (corresponding to P in the axis of the bore) is measured by the angle $P'TB$, where $P'T$ is a tangent to the curve at the point P'. In guns rifled with increasing twist, the angle of spiral at the muzzle is called the *final angle of rifling* and that at the breech end of the rifling is called the *initial angle of rifling*. PP' and BC represent the linear distances through which the exterior of the projectile has rotated when it has arrived at the points P and B in the axis of the bore respectively.

Twist of rifling. The *twist of rifling*, with *uniform* twist, is estimated by the distance (generally expressed in calibres) in which the spiral makes *one turn;*

with the *increasing* twist, it is estimated at any point by the distance in which the spiral would make *one turn* had the angle of spiral remained uniform, and the same as at the given point.

Relation between angle of spiral and twist of rifling.

Suppose α the angle of spiral at any point in the bore; also suppose the twist of rifling at the same point to be one turn in n calibres. Then

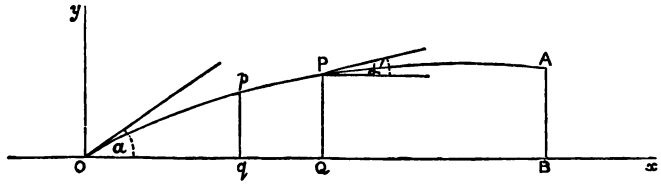
$$\tan \alpha = \frac{\pi}{n};^*$$

which expresses the relation between the angle of spiral and twist of rifling at any point in the bore.

Determination of the twist of rifling at any point in the bore.

PROBLEM.—To find the twist at any point of the bore of a gun rifled with increasing twist.

Let α and α' be the angles of spiral at the muzzle and at the breech respectively; n and m the number of calibres in which the spiral makes one turn at the muzzle and at the breech respectively.



In the figure, OP represents the curve (increasing twist)—O at the muzzle and P at the breech end of the rifling. The curve OP is a parabola, whose vertex is at A; and is referred to vertical and horizontal co-ordinates, whose origin is at O. Then

$$\tan \alpha = \frac{\pi}{n}, \text{ and } \tan \alpha' = \frac{\pi}{m}.$$

Let $y = bx + cx^2$ be the equation to the curve.

Differentiating,

$$\frac{dy}{dx} = b + 2cx; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where $x = 0$, $\frac{dy}{dx} = \tan \alpha = b$.

Let OQ (the length of rifling in the bore) $= l$. Then from equation (1)

$$\tan \alpha' = \tan \alpha + 2cl;$$

or

$$2c = \frac{\tan \alpha' - \tan \alpha}{l} = \frac{\frac{\pi}{m} - \frac{\pi}{n}}{l}.$$

* *Vide* "Treatise on the Construction and Manufacture of Ordnance in the British Service," p. 41.

Let $Oq = h$, and ϕ the angle of spiral at p. Then from equation (1)

$$\tan\phi = \tan\alpha + 2ch$$

$$= \frac{\pi}{n} + \frac{h\left(\frac{\pi}{m} - \frac{\pi}{n}\right)}{l} = \frac{\pi}{n}\left\{1 + \frac{h}{l}\left(\frac{n}{m} - 1\right)\right\}.$$

Therefore twist of rifling at the point p is

$$\text{one turn in } \frac{n}{1 + \frac{h}{l}\left(\frac{n}{m} - 1\right)} \text{ calibres.} \quad \ldots\ldots\ldots\ldots\ldots\ldots (A)$$

For example: in the 12-in. gun of 25 tons, the twist at the muzzle is one turn in 50 calibres; that at the breech is one turn in 100 calibres. The length of rifling is 127 ins. It is required to find the twist in calibres at 8 ins. from the muzzle.

Here $n = 50$,
$m = 100$,
$l = 127$ ins.
$h = 8$ ins.

Then, substituting in the above formula (A), the twist at 8 ins. from the muzzle is

$$\text{one turn in } \frac{50}{1 + \frac{8}{127}\left(\frac{50}{100} - 1\right)} = 51{\cdot}626 \text{ calibres.}$$

Determination of linear rotation of exterior of projectile, while passing down the bore.

Next, to find the linear distance the exterior of the projectile has rotated while passing down the bore.

Suppose $Oq = h$ (the distance passed down the bore by the projectile); then

$$pq = bh + ch^2 = h\tan\alpha + h^2\frac{\tan\alpha - \tan\alpha}{2l}$$

$$= \frac{\pi h}{n} + \frac{\pi\left(\frac{1}{m} - \frac{1}{n}\right)h^2}{2l}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (B)$$

For example: it is required to find the linear distance which the exterior of the projectile has rotated in the 12-in. gun of 25 tons, when it has passed down 8 ins. of the bore from the muzzle.

In equation (B),

$$h = 8 \text{ ins.}$$

and substituting as before, the linear distance of rotation of the exterior of the projectile at 8 ins. from the muzzle is

$$\frac{8\pi}{50} + \frac{\pi\left(\frac{1}{100} - \frac{1}{50}\right)}{254} \times 64 = \frac{4000\pi}{25400} = {\cdot}49 \text{ ins.}$$

In several Woolwich guns the twist is zero at the breech end of the rifling; so that in this case

$$\tan \alpha' = \frac{\pi}{m} = 0, \text{ or } \frac{1}{m} = 0.$$

So that from equation (*A*) the twist of rifling at a point in the bore at h inches from the muzzle is

$$\text{one turn in } \frac{n}{1 - \frac{h}{l}} = \frac{nl}{l - h} \text{ calibres;} \quad \ldots\ldots\ldots\ldots\ldots\ldots (C)$$

and from equation (*B*) the linear distance of rotation of the exterior of the projectile at a distance of h inches down the bore is

$$\frac{\pi h}{n} - \frac{\pi h^2}{2nl} = \frac{\pi h}{n}\left(1 - \frac{h}{2l}\right). \quad \ldots\ldots\ldots\ldots\ldots\ldots (D)$$

Examination of recovered projectiles for groove-marks on body and studs.

It is often important, when examining projectiles which have been recovered after firing, to note the marks of the grooves on their bodies and studs, in order to determine the exact position in the bore of the gun where these marks were impressed.

For example: an 11-in. common shell, after having been fired, was recovered and examined for marks of the grooves on the driving edges of the studs. The impression on the front stud was due to a twist of one turn in 385 ins., or one turn in 35 calibres—the same as at the muzzle of the gun; showing that the front stud was in *bearing* at the muzzle. The impression on the rear stud was due to a twist of one turn in 592 ins., or one turn in 53·8 calibres; and the impression of the groove marks on the body of the shell (owing to contact of the body of the shell with the gun, from defective centring), was due to a twist of one turn in 1584 ins., or one turn in 144 calibres.

From these data it can be ascertained at what distance from the muzzle the *rear* stud ceased to rotate the projectile, and the front stud came into bearing; and also at what distance from the muzzle the shell must have been when there was contact between the body of the shell and the bore of the gun.

The length of rifling in the 11-in. M.L. gun is 119 ins., and the twist starts from zero at the breech to one turn in 35 calibres at the muzzle.

The amount of twist shown at the last contact of the rear stud with the side of the groove was one turn in 53·8 calibres.

From formula (*C*)

$$\frac{nl}{l - h} = 53{\cdot}8 \text{ (where } l = 119 \text{ and } n = 35),$$

$$\text{or } 53{\cdot}8h = 2237{\cdot}2,$$

$$\text{or } h = 41{\cdot}5 \text{ ins. from the muzzle;}$$

which shows that the front stud came into bearing at 41·5 ins. from the muzzle.

In the same way, the place of contact between the body of the shell and the lands of the gun can be found. In this case

$$\frac{nl}{l-h} = 144,$$

or $h = 90{\cdot}07$ ins. from the muzzle;

i.e., 28·93 ins. from the breech end of rifling.

The motion of translation and rotation of a projectile fired from rifled ordnance.

When the motion of projectiles fired from rifled ordnance comes under consideration, it is necessary to distinguish between the *motion of translation* and the *motion of rotation*.

Motion of translation refers to the motion of a projectile travelling through the air; the motion of rotation to its spinning round its longer axis.

Both these motions are combined in the motion of an elongated projectile fired from rifled ordnance; but they are quite independent of one another, and may be considered separately.

Velocity of translation.

The *velocity of translation* of a projectile is usually measured by its *linear* velocity in feet per second.

Velocity of rotation.

The *velocity of rotation* of a projectile is usually measured by its *angular* velocity in units of circular measure per second. The angular velocity of a projectile making *one turn* in one second is 2π.

Now, if ω be the angular velocity of a projectile per second, and s the *number of turns* it makes in a second, then

$$s = \frac{\omega}{2\pi}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

i.e., the number of revolutions per second is proportional to the *angular velocity*.

Also, if the projectile makes *one revolution* in n calibres—*i.e.*, in nd inches, where d is the calibre of the gun in inches, and V the muzzle velocity in feet per second—then the number of revolutions of the projectile per second as it leaves the muzzle of the gun is

$$s = \frac{12V}{nd}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Equating (1) and (2), it will be seen that

$$\omega = \frac{24\pi V}{nd}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

or the angular velocity of a projectile is directly proportional to the muzzle velocity, and inversely proportional to the length of twist.

Linear velocity of rotation.

The linear velocity of rotation of any point in a rotating projectile is the *product* of its angular velocity into the distance of that point from the axis of rotation; or if ω be the angular velocity, and r the distance of the point from its axis of rotation in inches,

$$\text{linear velocity of rotation} = \omega r = \frac{2\pi Vr}{nd} \text{ in feet per second.}$$

Example.—To find the number of revolutions per second, the angular velocity, &c., of the 16-pr. shell at the muzzle. The muzzle velocity

of 16-pr. = 1358 f.s., calibre 3·6 ins., twist 1 turn in 30 calibres. From (2),

$$\text{number of revolutions per second} = \frac{12 \times 1358}{108} = 150{\cdot}9;$$

and from (3),

$$\text{angular velocity} = 2\pi \times 150{\cdot}9 = 301{\cdot}8\pi = 948{\cdot}1.$$

Also, linear velocity of rotation of a point on the surface of the projectile is

$$948{\cdot}1 \times 1{\cdot}77 = 1678{\cdot}2 \text{ ins. per second,}$$

since diameter of shell is 3·54 ins.; or, expressing the velocity in feet per second, the linear velocity of rotation of the 16-pr. shell

$$= \frac{1678{\cdot}2}{12} = 139{\cdot}8 \text{ ft. per second.}$$

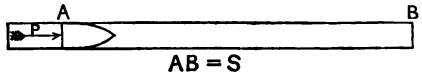
Work.—When a projectile has been set in motion from rest, some force or pressure must have been exerted upon it. This force or pressure is then said to have done *work;* and the amount of *work done* is measured by the product of the *pressure* into the *distance* through which the projectile has been moved. Work done is usually expressed in foot lbs., although for artillery purposes it is often expressed in foot tons. Work.

Thus, if P be the pressure in lbs. exerted on a projectile through the space of S feet, then the work done on the projectile is

$$P \times S \text{ ft. lbs.} = \frac{PS}{2240} \text{ ft. tons.}$$

But, in practice, the pressure exerted on a projectile during its passage through the bore of a gun is not constant but *variable;* so that to obtain the approximate amount of work done, the product of the *mean pressure* on the projectile into its *distance* moved through the bore of the gun must be taken. Then

P = mean pressure on projectile
= mean pressure on the bore of the gun.



Example.—If the total mean pressure on the base of a Palliser shell, fired with a battering charge out of the 10-in. M.L. gun, were 516 tons, what would be the work done on the shell at the muzzle?

The distance through which the pressure acts must be the difference between the length of the bore and the length of the cartridge.

Length of bore of 10-in. M.L. gun is 145·5 ins.
Mean length of cartridge..... " 25·5 "

The difference will be the distance passed over by the shell from its seat to the muzzle—viz., 120 ins. = 10 ft.; consequently, work done = 10 × 516 = 5160 ft. tons.

It is often convenient to express the pressure exerted on the bore of the gun in terms of the number of *square inches* in its sectional area, by Pressure per square inch on the bore of the gun.

dividing the mean pressure by the number of square inches in its sectional area. Thus

$$p = \text{pressure per square inch} = \frac{\text{mean pressure}}{\pi \cdot \frac{d^2}{4}} = \frac{P}{\cdot 7854 d^2};$$

where d is the diameter of the bore in inches.

Work done per square inch.

Similarly,

$$\text{work done per square inch} = \frac{\text{total work done}}{\cdot 7854 d^2} = \frac{PS}{\cdot 7854 d^2}.$$

Energy.

When a projectile is in motion, it is said to have *energy—i.e.*, it is capable of doing work, or overcoming resistance. The amount of energy is measured by the *product of the weight of the projectile into the height due to its velocity*, and is expressed in the same units as those of work done.

Thus, if W be the weight of the projectile in lbs., h the height it would require in vacuo to attain a velocity v in feet per second, then the energy of the projectile is

$$Wh = \frac{Wv^2}{2g} \text{ ft. lbs.};$$

where $h = \frac{v^2}{2g}$.*

Now, if V be the muzzle velocity of the projectile, its energy at the muzzle of the gun is $\frac{WV^2}{2g}$; and since the energy of the projectile is equivalent to the work done on it,

$$E = \frac{WV^2}{2g} = PS;$$

where E represents the energy at muzzle.

It will therefore be seen that the energy of the projectile at the *muzzle* is equal to the work done by the pressure of the powder gas in the bore of the gun.

EXAMPLE.—Find the energy of the 10-in. M.L. gun at the muzzle, firing Palliser shell with a battering charge. Weight of shell = 400 lbs., muzzle velocity = 1364 f.s.

$$\text{Energy at muzzle} = \frac{400 \times (1364)^2}{2 \times 32{\cdot}19} \text{ ft. lbs.}$$

$$= \frac{400 \times (1364)^2}{2 \times 32{\cdot}19 \times 2240} \text{ ft. tons} = 5160 \text{ ft. tons.}$$

i.e., if a uniform resistance of 5160 tons were opposed to the shell, it would penetrate *one foot* before it was brought to rest; or if the resistance were 516 tons, it would penetrate 10 ft., and so on.

Energy per inch of circumference.

In order to estimate the comparative power of guns for piercing iron plates, it is usual to express the energy of a projectile in terms of

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* *Vide* "Treatise on Dynamics,"—Motion of a Particle falling freely in vacuo.

the number of inches in its circumference—*i.e.*, by dividing the energy by the number of inches in the circumference of the projectile. Thus,

$$\text{energy per inch of circumference} = \frac{Wv^2}{4480g \times \pi d} \text{ ft. tons};$$

where d is the diameter of the projectile in inches.

Also, $$\text{energy per inch of circumference at muzzle} = \frac{E}{\pi d} = \frac{WV^2}{4480g \times \pi d};$$

thus, muzzle energy per inch of circumference of 10-in. Palliser shell

$$= \frac{5160}{3{\cdot}1416 \times 9{\cdot}92} = 165{\cdot}6 \text{ ft. tons.}$$

Energy due to translation.

This energy is energy due to the velocity of translation of the projectile. But the projectile has rotation round its longer axis; so that there is also energy due to the velocity of rotation, which, being small as compared to the energy due to translation, is usually neglected.

Energy due to rotation.

But in considering the rotation of projectiles, the energy due to rotation must also be taken into account.

If ω represent the angular velocity of the projectile, K its radius of gyration in feet, then

$$\text{energy due to rotation} = \frac{W(K\omega)^2}{2g}.$$

Total energy.

The *total energy* of a rifled projectile in motion

$$= \text{energy due to translation} + \text{energy due to rotation}$$

$$= \frac{W}{2g}v^2 + \frac{W}{2g}(K\omega)^2 = \frac{W}{2g}\{v^2 + (K\omega)^2\};$$

where W = weight of projectile and v its velocity.*

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* If V be the muzzle velocity, and ω the muzzle angular velocity, then

$$\text{total energy of projectile at muzzle} = \frac{W}{2g}(V^2 + K^2\omega^2);$$

but $$\omega = \frac{24\pi V}{nd} \text{ (see p. 37)},$$

and $$K = \frac{k}{12},$$

where k = radius of gyration in inches.

$$\therefore \text{ total energy at muzzle} = \frac{WV^2}{2g}\left\{1 + \left(\frac{2\pi}{nd}\right)^2 k^2\right\};$$

where n = twist of rifling in calibres, and d = diameter of projectile in inches.

It can be shown that the moment of inertia $\left(\frac{W}{g}k^2\right)$ of the head of a solid projectile with an ogival head of 1½ diameters

$$= \pi\rho d^5 \times {\cdot}015189, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (A)$$

or $$Mk^2 = \pi\rho d^5 \times {\cdot}015189;$$

where ρ is the density of the projectile, M is the mass of the projectile, and d its diameter.

Also, that the *volume* of the *ogival head*

$$= \pi d^3 \times {\cdot}15733. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (B)$$

EXAMPLE.—*To find moment of inertia of* 16-*pr. common shell* (*vide Changes in War Stores*, §2190):—

Ogival head. Body.

1st. Suppose solid.

(1) $M_1k_1^2 = \pi\rho d^5 \times {\cdot}01519 = \pi\rho\,(3{\cdot}54)^5 \times {\cdot}01519 = 8{\cdot}443 \times \pi\rho.$

(2) $M_2k_2^2 = \pi\rho\frac{d^4}{2^5} \times \text{length} = \pi\rho\frac{(3{\cdot}54)^4}{2^5} \times 6{\cdot}9 = 33{\cdot}87 \times \pi\rho.$

$\therefore$ total moment of inertia of solid projectile $= M_1k_1^2 + M_2k_2^2 = 42{\cdot}31\pi\rho.$ .....(1)

It must be remembered that the velocity of translation is given by the pressure of the powder gas on the base of the projectile, and that the velocity of rotation is given by the pressure of the sides of the grooves against the driving edges of the studs of the projectile, as it travels through the bore.

Pressure required to give rotation to projectiles.

The pressure on the driving edges of the studs depends on the angle of spiral in the gun, and also on the employment either of uniform or increasing twist, in order to give the requisite rotation to the projectile.

To compute moment of inertia of the core:—

(1) Head. $m_1\kappa_1^2 = \pi\rho\,(2{\cdot}34)^5 \times {\cdot}01519 = 1{\cdot}065\pi\rho,$

(2) Body. $m_2\kappa_2^2 = \pi\rho\,\dfrac{(2{\cdot}34)^4}{2^5} \times 6{\cdot}04 = 5{\cdot}659\pi\rho;$

$\therefore$ total moment of inertia of core $= m_1\kappa_1^2 + m_2\kappa_2^2 = 6{\cdot}724\pi\rho;$ .....(2)

$\therefore$ moment of inertia of empty shell $= 42{\cdot}31\pi\rho - 6{\cdot}724\pi\rho = 35{\cdot}59\pi\rho.$ .....(3)

If the shell is filled with water (the density of which does not differ greatly from that of the powder in a filled shell), then for the core, the density $\rho = 1$, and for the cast-iron shell $\rho = 7{\cdot}11$.

Therefore from (3),

moment of inertia of empty shell $= 35{\cdot}59 \times 7{\cdot}11\pi = 253{\cdot}0\pi,$

and from (2), " core filled with water $= 6{\cdot}7\pi,$

$\therefore$ total moment of inertia of filled shell $= 259{\cdot}7\pi.$ .....(4)

To find the radius of gyration of filled 16-pr. common shell:—

1st. Suppose solid.

(1) Ogival head. The volume of ogival head, or $\dfrac{M_1}{\rho} = \pi d^3 \times {\cdot}1573;$

$\therefore\ M_1 = \pi\rho d^3 \times {\cdot}1573 = \pi\rho\,(3{\cdot}54)^3 \times {\cdot}1573 = 6{\cdot}977\pi\rho.$

(2) Body. $M_2 = \pi\rho\left(\dfrac{d}{2}\right)^2 \times \text{length} = \pi\rho\left(\dfrac{3{\cdot}54}{2}\right)^2 \times 6{\cdot}9 = 21{\cdot}62\pi\rho.$

$\therefore\ M_1 + M_2 = 28{\cdot}60\pi\rho.$

2nd. To find $m_1 + m_2$ of core.

(1) Head. $m_1 = \pi\rho\,(2{\cdot}34)^3 \times {\cdot}1573 = 2{\cdot}015\pi\rho.$

(2) Body. $m_2 = \pi\rho\,(1{\cdot}17)^2 \times 6{\cdot}04 = 8{\cdot}268\pi\rho.$

$\therefore\ m_1 + m_2 = 10{\cdot}28\pi\rho.$

$\therefore$ mass of empty shell $= (28{\cdot}60 - 10{\cdot}28)\,\pi\rho = 18{\cdot}32\pi\rho$

$= 18{\cdot}32 \times 7{\cdot}11\pi = 130{\cdot}3\pi$ (since $\rho = 7{\cdot}11$),

mass of filled core $= 10{\cdot}28\pi$ (since $\rho = 1$),

$\therefore$ " filled shell $= (130{\cdot}3 + 10{\cdot}28)\pi = 140{\cdot}58\pi,$

$\therefore\ k^2 = \dfrac{\text{moment of inertia of filled shell}}{\text{mass of filled shell.}} = \dfrac{259{\cdot}7\pi}{140{\cdot}6\pi} = 1{\cdot}848$ ins.,

$\therefore\ k = \sqrt{1{\cdot}848} = 1{\cdot}35$ ins.;

where k is the radius of gyration of the filled shell.

Now, if the muzzle velocity of a filled 16-pr. common shell, fired with a service charge of 3 lbs. R.L.G. powder, is 1358 ft. per second, and weight of filled shell = 15 lbs. 13 ozs., the total energy at muzzle, or total work done by 3 lbs. of powder

$$= \frac{WV^2}{2g}\left\{1 + \left(\frac{2\pi}{nd}\right)^2 k^2\right\} = \frac{15{\cdot}81 \times (1358)^2}{2 \times 32{\cdot}19 \times 2240}\left\{1 + \left(\frac{2 \times 3{\cdot}1416}{30 \times 3{\cdot}54}\right)^2 1{\cdot}848\right\} \text{ ft. tons}$$

$$= 202{\cdot}2\,(1 + {\cdot}001894) \text{ ft. tons} = 202{\cdot}2 \times 1{\cdot}0019 = 202{\cdot}6 \text{ ft. tons.}$$

$\therefore$ work done by each lb. of powder in the charge $= \dfrac{202{\cdot}6}{3} = 67{\cdot}5$ ft. tons.

Since the work done in rotating the projectile is such a small per-centage of the work done in giving velocity, the former is usually neglected in calculations for penetration of iron plates, &c.

Captain Andrew Noble, of Elswick, has calculated the total pressure on the base and also on the studs of a 10-in. shell fired with a charge of 70 lbs. of pebble powder, in the case of the gun being rifled (1) with uniform twist, or (2) with increasing twist.*

The uniform twist is taken at one turn in 40 calibres; and the increasing twist commences with one turn in 100 calibres, and terminates at the muzzle with one turn in 40 calibres—the same as the uniform twist at the muzzle; so that the shell, while moving with the same velocity at the muzzle, would have the same muzzle velocity of rotation in both cases.

Travel of shell.	Total pressure on base of shell in both cases.	Corresponding velocity in both cases.	Total pressure on studs.	
			Uniform twist. (1)	Increasing twist. (2)
ft.	tons.	f.s.	tons.	tons.
·000	0	0	0·0	0·0
·333	1547	411	68·5	31·2
·945	1077	675	47·7	28·7
1·834	781	873	34·6	29·0
2·723	621	992	27·5	30·2
3·612	510	1078	22·6	31·4
4·500	424	1146	18·7	32·3
5·389	356	1200	15·8	33·0
6·278	305	1245	13·5	33·8
7·167	268	1282	11·8	34·5
8·055	240	1311	10·6	35·2
8·944	220	1333	9·7	35·8
9·833	205	1349	9·1	36·3

It appears from this table that the pressure on the studs at the commencement of motion is much less in the increasing than in the uniform twist, and consequently that the studs on the projectile would not be so severely strained, nor the driving sides of the grooves of the gun so much indented in the increasing as in the uniform twist. This is what is found to be the case in practice, and is the reason why the increasing twist has been preferred to the uniform twist for the heavier natures of guns.

The *velocity of recoil* is generally taken as the velocity imparted to the gun and carriage by the discharge of the piece. It is the greatest when the projectile leaves the muzzle. Velocity of recoil.

Let V'' be the maximum velocity of recoil,
v the muzzle velocity of projectile,
w the weight of projectile,
w' the weight of charge of powder,
W'' the weight of gun and carriage.

Then, if the momentum of the gun and carriage be taken equal to the momentum of the projectile,

$$W''V'' = (w + Cw')\,v,$$

$$\text{or } V'' = \frac{(w + Cw')\,v}{W''}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

* *Vide* "Proceedings, R.A. Institution," Vol. VIII. p. 365.

where C is a coefficient varying in different guns, which may be determined by experiment.

But this formula is not strictly correct, for three reasons :—

(1) On account of the difference between the sectional areas of the projectile and of the bore of the gun when there is *windage*.

(2) On account of the uncertainty of the amount of momentum communicated to the gun by the motion of the centre of gravity of the powder charge.

(3) On account of the small additional momentum communicated to the gun after the projectile has left the muzzle.

Since the sectional areas of the bore and projectile are as the squares of their diameters, if d be the diameter of the projectile and D that of the bore, the momentum communicated to the gun will be $\frac{D^2}{d^2} wv$; and if the small additional momentum communicated to the gun and carriage after the projectile has left the muzzle be expressed by $W''V'$, and if $Cw'v$ be the amount of momentum communicated to the gun and carriage by the motion of the centre of gravity of the powder charge, then

$$W''V'' = \frac{w}{d^2} D^2 v + Cw'v + W''V'. \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Gun pendulum.

It is the uncertainty of these terms that makes the measurement of muzzle velocities by means of the *gun pendulum* so inaccurate; but the instrument is well adapted to measure the velocity of recoil.

The gun pendulum consists of a frame in which a light gun is suspended by means of a rod. The pendulum, of which the gun forms the *bob* is capable of moving freely when the gun is fired. An arc graduated in degrees and minutes is attached to the frame, by means of which the angle of recoil is registered.

Great care is necessary, in adjusting the weights, to get the centre of percussion very nearly in the axis of the bore.*

Experiments on recoil.

Some experiments with a gun pendulum were made at the Royal Laboratory, Woolwich, in 1874, to determine the recoil of the Martini-Henry and Snider rifles. The rifles were alternately fixed in the pendulum with their barrels horizontal and weighted up to the same weight, and so adjusted that the centre of percussion was very nearly in the axis of their barrels. They were fired by means of a hair trigger, so as to avoid communicating any sensible motion to the pendulum in firing. The angle of recoil was then read off, and the mean angle of five rounds recorded, and the work done on the pendulum by the discharge of the rifle computed, and thence the work done in recoil. (*Vide* the following table.)

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* *Vide* Boxer's "Treatise on Artillery," p. 36.

Nature of rifle.	Weight of rifle.	Nature of powder.	Charge of powder.	Weight of bullet.	Muzzle velocity.	Work done in recoil.*
	lbs.		grs.	grs.	f.s.	ft.lbs.
Martini-Henry...	8·81	R.F.G.^{2}	85	480	1303	19·07
" ...	8·81	C. & H.	80	410	1380	15·43
Snider	8·95	R.F.G.	70	480	1248	16·39

* As determined by experiment with gun pendulum.

The muzzle velocities were taken by means of Professor Bashforth's gravity chronograph, and are all below the average, as they were taken in the winter, at the same time as the experiments on the recoil. The recoil of the Martini-Henry rifle with the service cartridge is 19·07 ft. lbs.; in other words, the recoil of the rifle would exert a steady pressure on the shoulder of 19·07 lbs. through the *space of one foot*, or $2 \times 19{\cdot}07 = 38{\cdot}14$ lbs. through the space of half a foot; or generally $n \times 19{\cdot}07$ lbs. through the space of $\frac{1}{n}^{\text{th}}$ of a foot—the intensity of the blow on the shoulder for the same amount of recoil depending on the skill of the man in firing.

In these experiments, as there is no windage, the error due to windage disappears; the small additional momentum communicated to the rifle after the bullet has left the muzzle may also be neglected; so that equation (1) may be used without sensible error, or

$$W''V'' = (w + Cw')\,v,$$

$$\text{or } V'' = \frac{(w + Cw')\,v}{W''}.$$

Now, the *energy of recoil* (which it must be remembered is entirely different from the momentum communicated by the discharge of the piece) is $\frac{W''V''^2}{2g}$; or, substituting for V'' its value, Energy of recoil.

$$\text{energy of recoil (or work done in recoil)} = \frac{W''V''^2}{2g} = \frac{W''}{2g}\left(\frac{w + Cw'}{W''}\right)^2 v^2;$$

and if the weight of the charge is $\frac{1}{n}^{\text{th}}$ the weight of the projectile,

$$w' = \frac{w}{n};$$

$$\text{energy of recoil (or work done in recoil)} = \frac{wv^2}{2g} \cdot \frac{w}{W''}\left(1 + \frac{C}{n}\right)^2. \quad \ldots\ldots(3)$$

The coefficient C varies with the nature of ordnance or small-arm, system of rifling, and also with the powder employed; but for the *same* description of gun or rifle fired under the *same* general conditions, the coefficient C may be taken as practically constant.

In the above experiments with the Martini-Henry rifle $C = 96$; and with the Snider $C = 75$.

If $\frac{C}{n}$ in equation (3) be assumed constant (which is very nearly the case when similar guns are fired with charges of powder having the same proportion to the weights of the projectiles), then

$$\text{energy of recoil} \propto \frac{wv^2}{2g} \times \frac{w}{W''},$$

or

$$\text{energy of recoil} \propto \text{muzzle energy of projectile} \times \frac{w}{W''};$$

so that if the muzzle energy of projectile is constant, the energy of recoil varies *directly* as the weight of the projectile and *inversely* as the weight of the gun and carriage. The object of the artillerist is clearly to make the muzzle energy of the projectile a maximum, and to reduce the energy of recoil as much as possible consistent with the requirements of the particular gun for *range* and *mobility*.

How the energy of recoil may be reduced.

The energy of recoil may be reduced either (1) by decreasing the weight of the projectile, (2) by decreasing the muzzle velocity, or (3) by increasing the weight of gun and carriage. But if the *weight of the projectile* be much decreased, the gun will have less range and penetration, although there will be the advantage of carrying more rounds of ammunition in the limber and wagon; if the *muzzle velocity* be much decreased, the trajectory will not be so flat, the accuracy of fire will not be so good (especially with shrapnel shell); if the *weight of the gun* and carriage be much increased, it becomes more difficult to move it from place to place, and consequently its mobility becomes impaired. When the recoil cannot thus be reduced to the required limit, it becomes necessary to resort to mechanical or other means for checking it.

The ratio of the weight of projectile to the weight of the gun and carriage is therefore determined by the nature of service for which it is to be employed—whether for *indirect* fire with low velocities, or for *direct* fire with high velocities—as well as the mobility required for the particular service.

Kemmis's table of service M.L.R. ordnance.

Major Kemmis's Table of Comparison of M.L.R. Guns gives the weight, length, and diameter of the cartridges and projectiles used with them; the ratio of the weights of their projectiles and charges; the work done on projectile and cartridge; the weight of gun and carriage; the velocity of recoil; the work done in recoil upon, or energy of recoil of, gun and carriage; and the muzzle velocity corresponding to the charge.

EXAMPLE.—Required, the velocity of recoil of the 9-pr. M.L. gun of 6 cwt.

On referring to the table, weight of 9-pr. shell = 9·06 lbs., weight of charge = 1·75 lbs., muzzle velocity = 1390 f.s., weight of gun and carriage packed = 2049 lbs. Then from equation (1)

$$V'' = \text{velocity of recoil} = \frac{(w + Cw')v}{W''};$$

		Average pressure per sq. in., $= p$.	$\frac{W}{p}$	Carriage. Weight, $= W'$.	$\frac{W'}{P}$	$\frac{W}{W'}$	Gun and Carriage. Weight, $= W''$.	Velocity of recoil, $= V''$.	Work done upon, $= \frac{W''.V''^2}{2g}$	Nature.	
		lbs.		lbs.			lbs.	ft. secs.	ft. lbs.		
FIELD.	7-pr.	6,697	$\frac{1}{33\cdot48}$	350	$\frac{1}{136\cdot20}$	$\frac{1}{1\cdot75}$	550	14·31	1,749	7-pr. of 200 lbs. (steel).	FIELD.
	9-pr.	9,705	$\frac{1}{14\cdot44}$	1,280	$\frac{1}{53\cdot58}$	$\frac{1}{1\cdot90}$	2,049[j]	7·33	1,711	9-pr. of 6 cwt.	
	9-pr.	10,130	$\frac{1}{14\cdot23}$	1,280	$\frac{1}{55\cdot93}$	$\frac{1}{1\cdot42}$	2,273[j]	6·60	1,538	9-pr. of 8 cwt.	
	16-pr.	10,730	$\frac{1}{9\cdot55}$	1,431	$\frac{1}{76\cdot26}$	$\frac{1}{1\cdot06}$	2,896[j]	8·86	3,534	16-pr. of 12 cwt.	
	25-pr.	9,109	$\frac{1}{4\cdot52}$	1,700	$\frac{1}{67\cdot32}$	$\frac{1}{\cdot84}$	3,755[j]	9·86	5,671	25-pr. of 18 cwt.	
	40-pr.	11,090	$\frac{1}{2\cdot91}$	3,460	$\frac{1}{56\cdot78}$	$\frac{1}{\cdot90}$	7,268	8·02	7,263	40-pr. of 34 cwt.	
	40-pr.	8,857	$\frac{1}{2\cdot25}$	3,460	$\frac{1}{45\cdot34}$	$\frac{1}{\cdot88}$	7,380	7·94	7,228	40-pr. of 35 cwt.	
	64-pr.	11,480	$\frac{1}{1\cdot60}$	3,715	$\frac{1}{96\cdot34}$	$\frac{1}{\cdot51}$	10,883	10·17	17,500	64-pr. of 64 cwt.	
	6·3-in.	6,751	$\frac{1}{1\cdot35}$	3,480	$\frac{1}{60\cdot46}$	$\frac{1}{1\cdot72}$	5,496	9·70	8,028	6·3-in. howr. of 18 cwt.[a]	
	8·0-in.	11,320	$\frac{1}{2\cdot19}$	4,802	$\frac{1}{118\cdot40}$	$\frac{1}{\cdot93}$	9,954	15·13	35,390	8·0-in. howr. of 46 cwt.	
GARRISON AND NAVAL.	64-pr.	6,757	$\frac{1}{1\cdot04}$	1,232[d]	$\frac{1}{170\cdot40}$	$\frac{1}{\cdot19}$	7,728	11·60	16,140	64-pr. of 58 cwt.	GARRISON AND NAVAL.
	64-pr.	6,947	$\frac{1}{\cdot87}$	1,596[e]	$\frac{1}{135\cdot20}$	$\frac{1}{\cdot30}$	9,548	9·27	12,760	64-pr. of 71 cwt.	
	64-pr.	9,942	$\frac{1}{1\cdot37}$	1,326[f]	$\frac{1}{231\cdot40}$	$\frac{1}{\cdot16}$	8,494	12·05	19,150	64-pr. of 64 cwt.	
	80-pr.	8,145	$\frac{1}{\cdot72}$	1,568[d]	$\frac{1}{161\cdot40}$	$\frac{1}{\cdot14}$	12,768	8·73	15,110	80-pr. of 5 tons.	
	7-in.	10,190	$\frac{1}{1\cdot01}$	2,716[f]	$\frac{1}{144\cdot30}$	$\frac{1}{\cdot26}$	12,796	12·77	32,410	7-in. of 90 cwt.	
	7-in.	18,500	$\frac{1}{1\cdot27}$	3,500[f]	$\frac{1}{203\cdot40}$	$\frac{1}{\cdot24}$	18,060	12·21	42,400	7-in. of 6½ tons.	
	7-in.	16,570	$\frac{1}{1\cdot05}$	3,052	$\frac{1}{208\cdot80}$	$\frac{1}{\cdot19}$	18,732	12·06	42,270	7-in. of 7 tons.	
	8-in.	16,440	$\frac{1}{\cdot81}$	4,788	$\frac{1}{172\cdot50}$	$\frac{1}{\cdot23}$	24,948	12·13	57,060	8-in. of 9 tons.	
	9-in.	17,280	$\frac{1}{\cdot64}$	4,732	$\frac{1}{232\cdot30}$	$\frac{1}{\cdot17}$	31,612	13·41	88,200	9-in. of 12 tons.	
	10-in.	17,250	$\frac{1}{\cdot42}$	7,504	$\frac{1}{180\cdot50}$	$\frac{1}{\cdot18}$	47,824	13·38	132,700	10-in. of 18 tons.	
	11-in.	17,530	$\frac{1}{\cdot31}$	7,504	$\frac{1}{222\cdot00}$	$\frac{1}{\cdot13}$	63,504	12·78	161,300	11-in. of 25 tons.	
	12-in.	15,520	$\frac{1}{\cdot27}$	7,504	$\frac{1}{233\cdot38}$	$\frac{1}{\cdot13}$	63,504	14·03	194,100	12-in. of 25 tons.	
	12-in.	16,730	$\frac{1}{\cdot21}$	13,328	$\frac{1}{141\cdot90}$	$\frac{1}{\cdot17}$	91,728	11·46	186,900	12-in. of 35 tons.	
	12·5-in.	17,040	$\frac{1}{\cdot20}$	13,328	$\frac{1}{157\cdot00}$	$\frac{1}{\cdot15}$	98,448	13·45	276,400	12·5-in. of 38 tons.	
	16-in.	19,540	$\frac{1}{\cdot11}$	96,544[g]	$\frac{1}{40\cdot68}$	$\frac{1}{\cdot53}$	275,744	11·33	550,300	16-in. of 80 tons.[a]	
	10-in.	9,046	$\frac{1}{\cdot67}$	7,140	$\frac{1}{99\cdot49}$	$\frac{1}{\cdot53}$	20,580	14·09	63,270	10-in. howr. of 6 tons.[a]	

[f] S.S. iron. [g] Travelling. [j] Packed.

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or taking $C = 1$, for purposes of comparison,

$$V'' = \frac{(w + w')v}{W''} = \frac{(9{\cdot}06 + 1{\cdot}75)\ 1390}{2049} = 7{\cdot}33 \text{ f.s.},$$

as in the table. Also,

$$\text{energy of recoil (or work done in recoil)} = \frac{W''V''^2}{2g} = \frac{2049 \times (7{\cdot}33)^2}{2 \times 32{\cdot}2} = 1711 \text{ ft. lbs.}$$

For destructive effect on the carriage caused by the shock of discharge of the gun, *vide* "Treatise on Military Carriages," pp. 30–33.

CHAPTER III.

Explosion and Pressure in the Bore of a Gun.

Explosion of Gunpowder.—Explosion of Gun-cotton.—Comparison of the Results of Explosion of R.L.G. and P. Powder.—Objects sought after by the use of Powder as a Propelling Agent.—Causes which affect the Time of Explosion of Powder.—Different Powders required to give the best Results in Light and Heavy Guns.—Effect of Explosion in the Bore of a Gun.—Experiments with the Noble Chronoscope.—Crusher Gauge.—Manner of Explosion of Powder in a Gun.—Resistance to the Motion of a Projectile in the Bore of a Gun.—Resistance to the Motion of Translation.—Resistance to the Motion of Rotation.—Resistance due to Friction.—The Maximum Limit of Pressure allowable in the Bore of Heavy Guns.

Explosion of gunpowder.

The principal advantage of gunpowder, over gun-cotton and other explosive substances for use with ordnance, is its comparatively slow explosion.

The progressive nature of the explosion of gunpowder allows of a certain interval of time to elapse before the maximum pressure of the gas is exerted in the bore of the gun. The projectile then has time to move, and consequently begins to move while the powder is exploding; so that an increasing space is provided for the rapidly expanding gas, which changes the character of the force exerted by the gas more into the nature of a pressure than an impulse.

An impulse is theoretically an indefinitely large force acting absolutely *instantaneously;* and although, strictly speaking, there are no forces in nature which answer to the above definition, yet there is a close approximation to it in the detonation of gun-cotton and other explosive substances.

Explosion of gun-cotton.

It is this approximation of the force of exploded gun-cotton to that of an impulse which renders it (in its present state) unfit for general use in the service of ordnance, straining as it does inordinately both projectile and gun.

Comparison of the results of explosion of R. L. G. and P. powder.

In quick-burning powders, such as R.L.G., a shorter interval of time elapses before the maximum pressure of the charge is exerted than in slow-burning powders like P. powder.

The *time of explosion* has an important effect on the maximum pressure in the powder-chamber. As the time of explosion increases, this pressure diminishes, and *vice versâ.*

From recent experiments with P. and R.L.G. powders in the bore of a 10-in. rifled M.L. gun of 18 tons, it was calculated that the time elapsed, before the maximum pressure was exerted in the bore, was with the (slow-burning) pebble powder ·00437 sec., and with the (quick-burning) R.L.G. powder ·00070 sec.* The maximum pressure of the P. powder was about 18 tons on the square inch, being considerably less than that exerted (22 tons) by the R.L.G. powder. (*Vide* Plate I.)

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* *Vide* "Researches on Fired Gunpowder," by Capt. A. Noble, F.R.S., and Professor Abel, F.R.S., p. 118.

Table showing the Results of Explosion of P. and R.L.G. Powder.

Charge of powder.	Weight of projectile.	Muzzle velocity.	Maximum pressure in the chamber.	Time of max. press.	Distance traversed in that time by projectile.
	lbs.	f.s.	tons per sq.in.	sec.	ins.
70 lbs. P.	300	1527	18	·00437	6·0
60 lbs. R.L.G.	300	1480	22	·00070	·6

The force developed, then, by fired R.L.G. powder approximates more to the nature of an impulse than that developed by fired P. powder, and consequently exerts a more destructive strain on both gun and projectile. It is for this reason that P. powder has been substituted for R.L.G. powder in the charges of the heavy rifled M.L. guns.

Objects sought after by the use of powder as a propelling agent.

The chief object to be attained by the use of powder as an explosive agent in the bore of a gun is to transmit the greatest amount of *muzzle energy* to the projectile with the least possible strain to the gun—*i.e.*, to transmit the maximum of *total pressure* to the base of the projectile, throughout the bore of the gun, with the minimum of *local pressure* to the powder-chamber.

Causes which affect the time of explosion of powder.

The *time of explosion* of powder depends principally on the size and shape of the grains, their density and hardness, the amount of glazing given to them, and the quantity of moisture contained in the powder.

Suppose a powder similar in all respects except in the size and shape of the grains; then the time of explosion, or conversion of the powder into gas, depends on the *rate of ignition* of the grains, and the *time of combustion* of each grain.

The rate of ignition of the grains depends on the facility with which the flame can penetrate to contiguous grains—*i.e.*, on the magnitude of the interstices between the grains, depending principally on the shape of the grains; while the time of combustion depends on the bulk or size of the grains themselves—*i.e.*, the smaller the *size* the quicker the combustion. These conditions (more fully explained in a treatise on gunpowder),* are to a certain extent opposed to one another; so that it will readily be understood that there is some shape and size of grain which would render the explosion the quickest possible.

A charge made up of large-grain powder explodes more slowly than that made up of a small-grain powder; for although, on account of the larger interstices between the grains of the former, ignition is more rapid, yet the combustion of each grain is slower than in the latter case. The former is called a slow-burning powder (comparatively), the latter a quick-burning powder. It follows, then, that in order to secure uniform results with similar charges of powder, the size and shape of the grains must be as nearly as possible the same.

The time of explosion is also affected by the density of the powder.

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* *Vide* "Short Notes on Gunpowder, and its Manufacture," by Major Wardell, R.A.

The denser the grain the more powder there is in it for a given size, and consequently the time of combustion is longer. Other things being equal, the denser the powder the slower burning it is, and the less the pressure in the bore of the gun.

The amount of moisture in the powder has the effect of modifying its time of explosion. The accompanying table shows the effect of moisture in powder: as the per-centage of moisture increases, the time of explosion is lengthened, and the pressure in the gun and the muzzle velocity of the projectile decreased.

Table showing the Effect of Moisture in Powder on Muzzle Velocity and Pressure in the Gun.

Sample of powder.	Per-centage of moisture.	Muzzle velocity in ft. secs.	Maximum pressure in the bore (ft. tons).
Pebble, ⅝ in....	·8	1537	21·38
	1·0	1524	20·18
	1·2	1512	19·12
	1·4	1502	18·18

The glazing and hardness of the grains retard the ignition of the powder, and consequently increase the time of explosion.

The time of explosion of powder in a *gun* is further affected by the form of cartridge, diameter of the powder-chamber, amount of air-space, and the point of ignition of the cartridge. (*Vide* Chapter IV.)

With large charges for heavy guns, it becomes especially important, in order to keep down the pressure within limits of safety, to increase the density and size of grain. The time of explosion is thus increased, and for a given charge both the muzzle velocity of projectile and the maximum pressure in the bore are decreased.

Table showing Density and Size of Grain of Powders used with Rifled Ordnance.

Description of powder.	Size of grain.	Mean density.
R.L.G.	4 to 8 meshes to an inch.	1·68
Pebble (P.)	⅝-in. cubes.	1·80
Cubical (P.^{2})	1¼-in. cubes.	1·78

Cubical powder (P.^{2}) was introduced for employment in the very large charges used with the 38 and 80-ton guns; since with ordinary pebble (P.) powder, owing to the effect of the *heat* and *pressure* developed in the partial explosion of such large charges, the time of explosion would be diminished, and the local pressure in the powder-chamber would exceed safe limits.

R.L.G POWDERS.

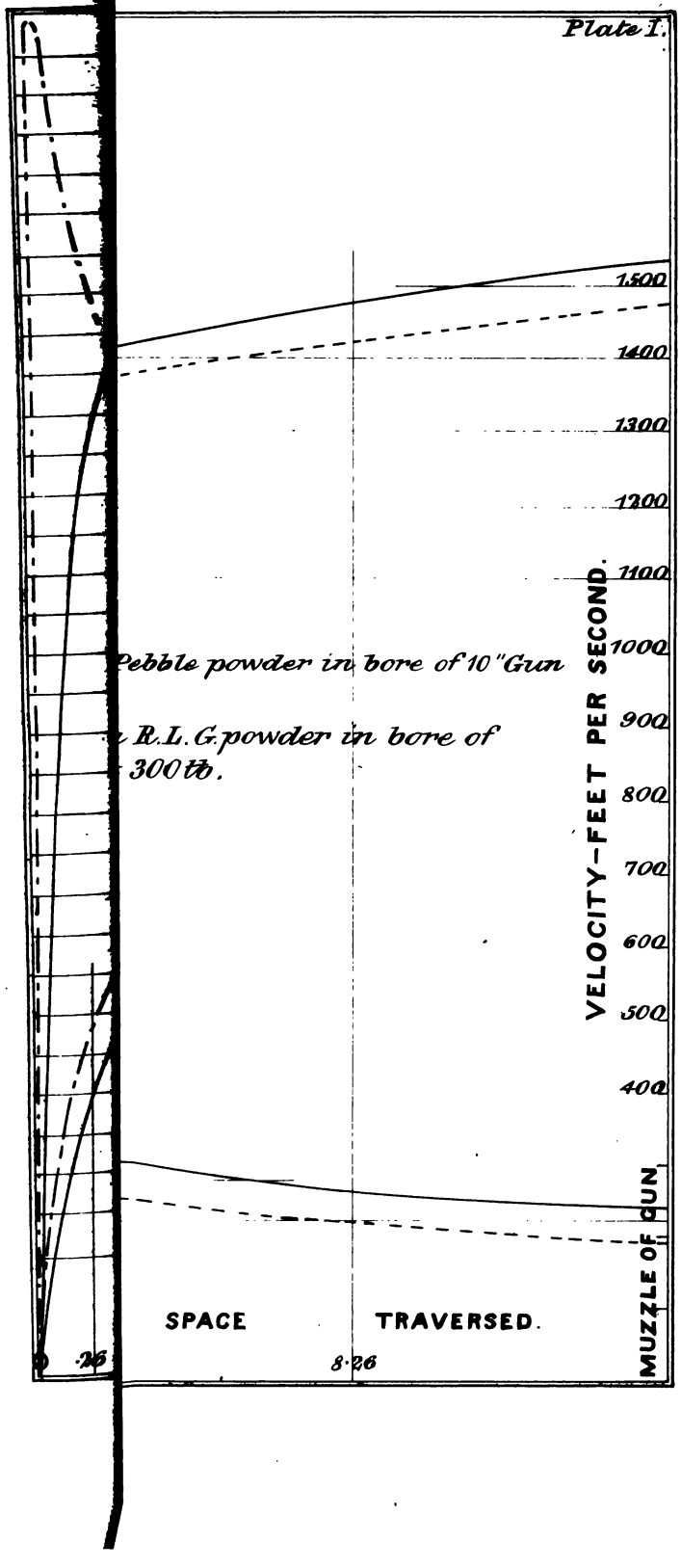
Plate I.
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
Pebble powder in bore of 10" Gun
R.L.G. powder in bore of
300 lb.
VELOCITY-FEET PER SECOND.
MUZZLE OF GUN.
SPACE TRAVERSED.
8·26
8·26

In different natures of ordnance, where the charge and calibre vary considerably, great attention must be paid to the regulation of the size and density of the grains of the powder employed, in order to transmit the maximum of muzzle energy to the projectile with the minimum of local pressure on the gun. A light projectile moves easily, and a quick-burning powder is found most suitable; a heavy projectile moves less easily, and a slow-burning powder is found most suitable.

Different powders required to give the best results in light and heavy guns.

When a charge of powder is exploded in a gun, a large quantity of gas is generated, which rapidly expands, and exerts an enormous pressure both on the chamber of the gun and on the base of the projectile. This pressure is usually measured in foot tons on the square inch. (*Vide* p. 13.)

Effect of explosion in the bore of a gun.

The pressure on the base of the projectile continually varies while it is in the bore of the gun (*vide* Pressure Curves, Plate I.), reaching a maximum a short time after the projectile has moved; then it gradually decreases towards the muzzle. The effect of this pressure on the base of the projectile is to give *velocity* to the projectile, which continually increases towards the muzzle, where it is the greatest, and it is then called the muzzle velocity.

There is a corresponding pressure on the chamber and bore of the gun; as at any given instant of explosion the pressure of the expanding gases may be supposed equal in every direction—*i.e.*, on the base of the projectile and on the walls of the bore. The pressure of fired gunpowder in the bores of guns has been very fully investigated by the Committee on Explosives, under the presidency of Colonel Younghusband, R.A., F.R.S.; and details of experiments are given in the Progress Reports published from time to time by that Committee. By means of the Noble chronoscope and the crusher gauge (for details of which *vide* App. I.), the pressures in a gun at any part of the bore, and the corresponding velocities of the projectile in the bore, can be ascertained.

Experiments with the Noble chronoscope.

The Noble chronoscope measures the time occupied by the projectile in passing over certain known intervals in the bore of the gun. The velocity of the projectile at any part of the bore is thus determined, and thence the pressures necessary to produce such velocities are calculated.

The results are plotted off on a diagram to scale. (*Vide* Plate I.)* The horizontal distances at the bottom of the plate represent the space in feet traversed from the seat of the projectile to the muzzle of the gun. The left-hand vertical column represents a scale of pressure in tons per square inch, or corresponding pressure in atmospheres; the right-hand vertical column represents a scale of velocity in feet per second.

Plate I. shows the result of an experiment made with pebble and R.L.G. powder to determine the pressure and velocity given by them respectively in the bore of a 10-in. M.L. gun, when firing a 300-lb.

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* *Vide* "Researches on Fired Gunpowder," Plate XIX.

projectile with battering charges—*i.e.*, with 70 lbs. P., or 60 lbs. R.L.G.*

The continued curves show the results with P. powder, and the dotted curves those with R.L.G. powder. From this diagram the pressure and velocity may be seen by inspection at any part of the bore of the gun. By comparing the pressure curves, the advantage of P. over R.L.G. powder is well shown. The maximum pressure in the gun in each case is represented by the vertex of the curve, and the work done on the projectile is represented by the respective areas enclosed by the curves. The area of the P. pressure curve is greater than that of the R.L.G. pressure curve, while the vertex is lower; which indicates that the muzzle energy of the projectile is greater, and that the maximum pressure in the gun is less, when the P. powder charge is fired instead of the charge of R.L.G. powder.

Crusher gauge.

Another method of determining the pressure in the bore of a gun is by means of the crusher gauge. (*Vide* App. I.) This is done by direct measurement, and is a rougher method of approximating to the true pressure than that by the chronoscope. However, the pressures calculated from chronoscopic experiments and those observed directly by means of the crusher gauge agree very closely, when slow-burning pebble powder is used; but when a quick-burning powder like R.L.G. is used, the agreement is not so close, owing to the intense local action which is set up in the combustion of such powders, which produces anomalous local pressure in the chamber of the gun.†

Generally speaking, then, the chronoscope measures the *mean pressure* and the crusher gauge the *local pressure* in the powder-chamber; and by the combined use of both these methods of experimenting, practical results of the highest utility have been arrived at.

Manner of explosion of powder in a gun.

When a gun is fired, the charge of powder is very rapidly exploded, and *practically* (so far as useful effect is obtained from its combustion) explosion may be supposed to be completed when the *maximum pressure* is obtained. Some portion of the charge, no doubt, is ignited during the after passage of the projectile to the muzzle, and is blown out of the gun only partially exploded, more especially when battering charges of P. powder are fired; but the useful effect of this after combustion is only a small per-centage of the whole. It must not be supposed that the powder does comparatively little work after the maximum pressure has been attained: it is quite the contrary; but only that, practically, explosion is completed in a very short space of time. An inspection of the diagram (Plate I.) shows the general conditions of explosion very clearly. The maximum pressure in the powder-chamber was in this instance attained with R.L.G. powder when the projectile had moved $\frac{6}{10}^{\text{ths}}$ of an inch; with P. powder when the projectile had moved 6 ins. The powder, then, is not *instantaneously* exploded; but supposing the explosion to be practically completed when the maximum pressure is reached, it will be seen by

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* *Vide* Table showing Results of Explosion, &c., of P. and R.L.G. (p. 23).

† For information on this point, *vide* a paper by Major Morgan, R.A., late Assistant Superintendent Royal Gunpowder Factory, Waltham Abbey, on "The Determination of the Explosive Force of Gunpowder." ("Proceedings, R.A. Institution," Vol. VII. p 430.)

reference to the Table showing Results of Explosion, &c. (p. 23), that the explosion of 60 lbs. R.L.G. powder takes place in about $\frac{1}{6}^{th}$ of the time taken to explode 70 lbs. of P. powder under similar conditions.

It will be seen from the diagram (Plate I.) that the pressure due to explosion increases very rapidly to a maximum, then at first it rapidly, and afterwards more gradually, decreases towards the muzzle; owing to the continually increasing space for expansion of the gas, caused by the continually increasing forward velocity of the projectile.

Resistance to the motion of a projectile in the bore of a gun.

The effect of the pressure in the bore is to overcome the resistance to motion of the projectile, and to communicate velocity to the projectile. The resistances to the motion of a projectile in the bore of a gun must then be considered.

They may be classified thus :—

(1) The resistance to the motion of *translation* of the projectile.
(2) The resistance to the motion of *rotation* of the projectile.
(3) The *frictional* resistances of the studs and gas-check, opposing the motions of translation and rotation.

Resistance to the motion of translation.

The resistance to the motion of translation of the projectile depends on its weight and the rate of increase of its velocity conjointly. Thus

$$R \propto Wf;$$

where R denotes resistance, W weight of projectile, f the acceleration of the projectile* (or the rapidity with which velocity is communicated to it). The acceleration depends on the rapidity of explosion; and consequently the resistance to motion for the same weight of projectile would be greater in the case of a quick-burning than with a slow-burning powder. This would intensify the pressure in the powder-chamber, as has been shown.

A small difference in the weight of the projectiles, when a slow-burning powder is used, only increases slightly the maximum pressure in the bore. In experiments with a 12-in. gun of 38-tons, the Committee on Explosives found that the effect of increasing the weight of the shot from 700 lbs. to 800 lbs., while firing the same charge of P. powder, did not appear sensibly to increase the maximum pressure in the bore.† Hence it becomes possible to fire very heavy projectiles without increasing inordinately the pressure in the gun, by the employment of very slow-burning powder.

Resistance to the motion of rotation.

The resistance to the motion of rotation of the projectile is only a small per-centage of the resistance to the motion of translation. (*Vide* p. 15). This subject has been very fully discussed in the pages of "Proceedings, R.A. Institution" (Vol. VIII. p. 359), by Captain A. Noble, F.R.S., in a paper "On the Pressure required to give Rotation to Rifled Projectiles." Captain Noble states that in the 10-in. gun

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* The acceleration of the projectile in the bore (or the rate of increase of its velocity in a given time) is variable, owing to the variable pressure of the powder gas, and the varying resistances which are opposed to the motion of the projectile.

† *Vide* "Report on the Experimental Rifled M.L. Gun of 38 tons."

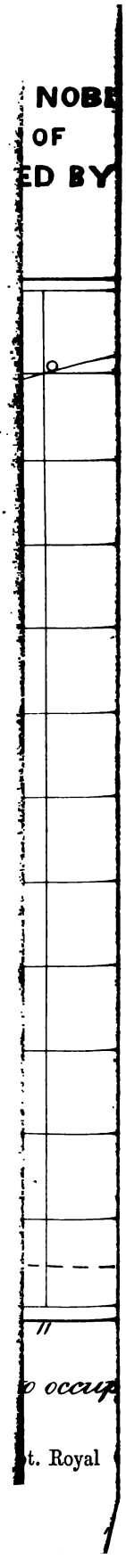
(and other guns similarly rifled), it is about 2¼ per cent. The increase of pressure in the bore tending to burst the gun, due to the resistance to the motion of rotation of the projectile, is very small: no sensible difference of pressure could be detected when the 10-in. gun was fired under similar conditions of charge and projectile, whether rifled or not —*i.e.*, whether rotation was imparted to the projectile or not.

Resistance due to friction.

The resistance due to friction with the B.L. Armstrong guns is considerable—the friction extending over the whole of the surface of the projectile which is in contact with the bore; but in the service M.L. guns it principally arises from the friction between the edge of the studs and the driving surface of the grooves. With the M.L. guns, the projectiles are in most cases approximately centred; so that there is little or no friction between the plane surfaces of the studs and grooves. The coefficient of friction between the edge of the studs and the driving surface of the grooves may be taken to be about ·17.

The maximum limit of pressure allowable in the bore of heavy guns.

In experimenting with powders of different shapes and densities, it has been considered desirable to assign a certain limit to the maximum admissible pressure in the powder-chamber, so as to avoid any undue strain on the material of the inner tube of the gun when repeatedly fired. The extreme limit is 25 tons on the square inch. The battering charges of the heavy Woolwich guns are regulated so as not to exceed this limit. With the 38 and 80-ton guns, it is considered desirable that the limit should not exceed 20 tons on the square inch.



Work done by powder.

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Resistance due to friction.

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The maximum limit of pressure allowable in the bore of heavy guns.

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CHAPTER IV.

Work Done by a Charge of Powder in connection with Muzzle Velocity and Pressure in the Bore.

Work done by Powder.—The Velocity of a Projectile at any point in the Bore of a Gun.—Muzzle Velocity.—Approximation to the Muzzle Velocity of a Gun by means of Noble's Tables.—Circumstances affecting Muzzle Velocity.—Charge of Powder.—Nature of Powder.—Weight of Projectile.—Length of Bore.—Calibre.—Diameter of Chamber, and Cubic Space occupied by the Charge.—Windage.—Position of the Vent.—Twist of Rifling.

Work done by powder.

Comparatively little was known of the work done by the explosion of gunpowder until it was made the subject of enquiry by Captain A. Noble, late R.A., F.R.S., and Professor Abel, F.R.S. Their researches have thrown much light on the subject. The results of their experiments are expressed by connecting the number of volumes of expansion of the powder charge in the bore of the gun with the maximum work per pound of powder which the powder is capable of realising.

The amount of work capable of being done on a projectile by the charge of powder in a gun depends principally upon the ratio of the cubic space occupied by the charge to the cubic capacity of the bore of the gun. This is *modified* in practice—

(1) By the nature of the powder—its density, hardness, &c., and the size and shape of its grains.

(2) The method of ignition adopted, and the position of the vent.

(3) The amount of air-space allowed. (*Vide* p. 36.)

(4) The weight of the projectile, its resistance to motion (both of translation and rotation), frictional and other resistances—whether due to compression (as in the Armstrong B.L. system), or to expansion (as in the polygroove M.L. system), or to the gas-checks used with Woolwich guns.

(5) The quantity of heat absorbed by the gun.

The annexed table is taken from the results obtained by Noble and Abel. The curve deduced from their experiments is given in Plate II.

A pound of powder is taken to occupy a space of 27·7 cub. ins.

The number of volumes of expansion of the powder charge in the bore of a gun is obtained by dividing the cubic capacity of the bore of the gun by the cubic space actually occupied by the charge under the above conditions.

Table showing Work Done per pound of Powder in a Gun.

Number of volumes of expansion in the bore.	Total work the powder is capable of realising per pound burnt, in ft. tons.	diff.	Number of volumes of expansion in the bore.	Total work the powder is capable of realising per pound burnt, in ft. tons.	diff.
5·0	83·53		7·6	97·96	
		72			43
5·1	84·25		7·7	98·39	
		70			42
5·2	84·95		7·8	98·81	
		69			42
5·3	85·64		7·9	99·23	
		67			41
5·4	86·31		8·0	99·64	
		65			40
5·5	86·96		8·1	100·04	
		64			40
5·6	87·60		8·2	100·44	
		62			40
5·7	88·22		8·3	100·84	
		61			39
5·8	88·83		8·4	101·23	
		60			39
5·9	89·43		8·5	101·62	
		59			38
6·0	90·02		8·6	102·00	
		57			38
6·1	90·59		8·7	102·38	
		55			37
6·2	91·14		8·8	102·75	
		55			37
6·3	91·69		8·9	103·12	
		54			36
6·4	92·23		9·0	103·48	
		53			36
6·5	92·76		9·1	103·84	
		52			35
6·6	93·28		9·2	104·19	
		50			35
6·7	93·78		9·3	104·54	
		50			35
6·8	94·28		9·4	104·89	
		49			34
6·9	94·77		9·5	105·23	
		48			34
7·0	95·25		9·6	105·57	
		47			33
7·1	95·72		9·7	105·90	
		46			33
7·2	96·18		9·8	106·23	
		45			32
7·3	96·63		9·9	106·55	
		45			32
7·4	97·08		10·0	106·87	
		44			
7·5	97·52				
		44			

By means of the above table, the maximum work that can possibly be obtained in any gun by a given charge of powder may be calculated.

EXAMPLE.—What is the maximum work that can possibly be done by 50 lbs. of P. powder in the 9-in. M.L. gun? (The length of the bore is 125 ins.)

$$\text{Volume of bore} = \frac{\pi}{4} \times 81 \times 125 \text{ cub. ins.},$$

$$\text{volume of charge (50 lbs.)} = 50 \times 27{\cdot}7 \text{ cub. ins.};$$

so that the number of the volumes of expansion of the powder charge contained in the bore of the gun are

$$\frac{\pi \times 81 \times 125}{200 \times 27{\cdot}7} = 5{\cdot}74.$$

From the table, then (by proportional parts), it will be seen that the *maximum* work capable of being performed by the charge of 50 lbs. of P. powder in the 9-in. M.L. gun

$$= 50 \times 88{\cdot}46 = 4423 \text{ ft. tons.}$$

But the actual muzzle velocity of the 9-in. gun, with a 250-lb. projectile and 50 lbs. of P. powder, is about 1420 f.s.; so that the muzzle energy communicated to the projectile is

$$\frac{250 \times (1420)^2}{4480 \times 32{\cdot}2} = 3494 \text{ ft. tons.}$$

A comparison of this with the maximum work that could theoretically be effected by the charge, shows that there is a deficiency of 929 ft. tons of energy not realised in practice. This is due to the *modifications* mentioned above. The greater the weight of the projectile, the more rapid the explosion of the powder, and the less the proportionate amount of heat absorbed by the gun, the more nearly will the maximum theoretical effect of the powder be realised. With service conditions, the heavier the gun the less the proportionate amount of heat absorbed. In the 9-in. gun the per-centage of maximum effect actually realised is

$$\frac{349400}{4423} = 79{\cdot}0.$$

This is called the *factor of effect* for this particular gun and charge.

Captain A. Noble has calculated the factor of effect for every gun and charge in the British service. A table of factors of effect is given in p. 354 of the "Treatise on the Construction and Manufacture of Ordnance in the British Service."

The velocity of a projectile at any point of the bore depends on its weight, and upon the work done on it up to *that point*. By the rule of work, it follows that

The velocity of a projectile at any point in the bore of a gun.

$$\frac{Wv^2}{2g} = kC;$$

where W = weight of projectile,
v = velocity at any point of the bore,
k = work done per pound of powder on projectile up to that point,
C = weight of charge in lbs.;

or
$$v = \sqrt{2g\frac{kC}{W}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

So that if the work done on a projectile at any point of the bore of a gun is known, its velocity at that point may be determined by formula (1); which, expressed in words, means that the velocity of a projectile at any point of the bore of the gun varies *directly* as the square root of the work done upon it up to that point, and *inversely* as the square root of its weight.

Consequently, if V represent the muzzle velocity of the projectile, and K the work done on it per pound of powder up to the *muzzle*,

Muzzle velocity.

$$V = \sqrt{2g\frac{KC}{W}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Now, if the work done on a projectile (KC) in a gun is constant, *the muzzle velocity of the projectile varies inversely as the square root of its weight;*

or
$$V \propto \sqrt{\frac{1}{W}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

But the work done on a projectile by a given charge of powder in the same or similar guns is *slightly increased* when the weight of the projectile is increased. For example: in the 10-in. M.L. gun, with a

charge of 60 lbs. R.L.G. powder, the work done by the powder is increased 5 per cent. when the weight of the projectile is doubled; with a charge of 70 lbs. P. powder, the work done by the powder is increased $8\frac{1}{2}$ per cent. when the weight of the projectile is doubled. In a 3-in. M.L. gun, with a charge of 2 lbs. R.L.G. powder, the work done by the powder was increased 14 per cent. when the weight of the projectile was doubled.

It is evident that when the weight of the projectile is increased, it takes a slightly longer time to move through the bore; so that more time is given for the pressure to act, and consequently a greater useful effect is obtained from the charge, and the factor of effect is increased.

Approximately correct muzzle velocities may be calculated on this principle for projectiles differing slightly in weight when fired from similar rifled guns with the same charge. Thus, if the weight (W) and muzzle velocity (V) of a projectile be known with a particular gun, then the muzzle velocity (V') of another projectile of known weight (W') fired from a similar gun with the same charge may be approximately found by the proportion

$$V' : V :: \sqrt{\frac{1}{W'}} : \sqrt{\frac{1}{W}},$$

or

$$V' = V\sqrt{\frac{W}{W'}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

It will be seen from formula (2) that if K (the work done per pound of powder) were constant,

$$V \propto \sqrt{\frac{C}{W}}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

or the muzzle velocity of a gun would vary directly as the square root of the weight of its charge, and inversely as the square root of the weight of its projectile. But it has been shown that the work done per pound of powder depends on the number of volumes of expansion of the powder charge contained in the bore of the gun; so that this formula is of no practical use for the calculation of muzzle velocity when the weights of charges vary in the same gun, because the smaller charge has more expansions than the larger when fired in the same gun.

Approximation to the muzzle velocity of a gun, by means of Noble's tables.

A more practical way of approximating to the muzzle velocity of a gun, when the weight of charge and projectile are known, is as follows, by means of Noble's table (p. 30):—

EXAMPLE.—Find approximately the muzzle velocity of the 38-ton gun of 12·5 ins., when firing an 800-lb. projectile with 110 lbs. of P. powder. (Length of bore 198 ins.)

The number of volumes of expansion of charge

$$= \frac{\pi \times (12{\cdot}5)^2 \times 198}{4 \times 110 \times 27{\cdot}7} = 7{\cdot}97,$$

then by the table (p. 30) the maximum work capable of being performed by the charge is

$$110 \times 99{\cdot}52 = 10947{\cdot}2 \text{ ft. tons.}$$

Allowing a small per-centage for loss of energy, due to the communication of heat, &c., &c., to the gun—which can only be judged by experience—a very close approximation to the muzzle velocity may be obtained. In this case, deduct 5 per cent. from the maximum theoretic work, in order to approximate to the work actually realised, which would then be 10,400 ft. tons. Substituting in formula (2),

$$\text{muzzle velocity} = \sqrt{2 \times 32{\cdot}2 \frac{10400 \times 2240}{800}} = 1369 \text{ f.s.};$$

which is very nearly the muzzle velocity determined by experiment.

The per-centage allowed in any case depends on the modifications alluded to in p. 29—viz., nature of the powder, method of ignition adopted, and the weight of the projectile, besides the amount of air-space allowed for the cartridge—which will be considered in due course. (*Vide* p. 36.)

Circumstances affecting muzzle velocity.

Circumstances which affect the muzzle velocities of projectiles fired out of rifled ordnance may be classified thus:—

Charge of powder.	Diameter of chamber.
Nature of powder.	Cubic space occupied by charge.
Weight of projectile.	Windage.
Length of bore.	Position of vent.
Calibre.	Twist of rifling.

Charge of powder.

By the employment of small charges of powder, the muzzle velocity of the projectile may be as low as desirable. The effect of increasing the charge by equal increments, is to increase the muzzle velocity rapidly at first (*vide* table), then less rapidly—depending on the nature of the powder employed and the length and calibre of the gun.

*9-pr. R.M.L. of 8 cwt.**

Charge.	Weight of shot.	M.V.
lbs.	lbs.	f.s.
1·75	9	1344
2·0	"	1356
2·25	"	1432
2·5	"	1457
2·75	"	1453
2·0	12	1196
2·25	"	1257
2·5	"	1287
2·75	"	1284
3·0	"	1259
Length of bore, 63·5 ins. Calibre ..... 3 "		

By a continued increase of the charge, the muzzle velocity continues to increase slowly until a point is reached (seldom or never attained in practice) when the muzzle velocity would be the greatest possible under the circumstances. Any further increase of the charge would then have the effect of diminishing the velocity until the bore was nearly full of powder, when the velocity would be very slight. It is evident that the larger the charge, the longer the cartridge must be; so that after a *certain limit is reached,* the explosion of the charge becomes incomplete, and the consequently reduced pressure acts on the projectile through an increasingly shorter distance.

* Experiments carried on in the Royal Gun Factories, Woolwich, 1873, by Major Maitland, R.A.

Nature of powder. The nature of powder employed exercises a considerable influence on the muzzle velocity. With guns firing comparatively small charges, the highest velocity is obtained by using a quick-burning powder—such as R.L.G. powder; but with guns firing large charges, the highest velocity (consistent with safety) is obtained by using a slow-burning powder—such as cubical pebble.

Weight of projectile. It will be seen from what has been stated (p. 31), that, for a given charge, the muzzle velocity of the projectile varies *approximately* inversely as the square root of its weight.

Length of bore. The internal space for the expansion of the powder charge in a gun of given calibre depends on the length of bore, and increases proportionately with the length of bore. Since the useful effect of the charge of powder depends (*vide* table, p. 30) on the number of expansions of the powder charge in the bore of the gun, it is evident that more work is done, and consequently a greater muzzle velocity communicated to the projectile, by a given charge of powder in a longer than in a shorter bore. Practically, the length of bore is limited by the necessary limitation of the weight of the gun for its particular service, and in field and siege guns by travelling conditions; so that it is not advisable to exceed certain limits.

Lengthening the bore of a gun not only gives a higher muzzle velocity for the same internal strain on the chamber, but permits of the use of a slower-burning powder; causing less strain on the gun and carriage—which latter is a point of great importance, not only with heavy guns, but also in connection with field and siege artillery equipment. The strain on the interior of the gun is communicated through the trunnions and elevating screw to the carriage; so that the less the maximum strain exerted on the gun, the less will be the maximum strain exerted on the carriage. The object, then, to be attained is to exert a low maximum pressure throughout a long bore, in order to give the projectile a high muzzle energy with as little destructive effect as possible on the gun and carriage. The destructive effect on the carriage depends in a great measure on the maximum pressure in the gun, and will be less in proportion as a lower maximum pressure is obtained in the gun. It is not meant that the work done in recoil exerts no destructive effect on the carriage, but that for a given muzzle energy of the projectile this necessarily destructive effect may be very much modified by the employment of a comparatively slower-burning powder and a long bore.*

The Committee on Explosives experimented on the original gun of 12 ins., of 35 tons, by lengthening the bore 3 ft. On firing the same charge of powder, it was found that the additional length of bore "yielded a gain in muzzle velocity of about 60 f.s., or nearly 5 per cent., on that obtained with the shorter gun."†

Calibre. As the calibre is increased, the powder charge is burnt more profitably—*i.e.*, more useful effect is obtained from the powder, and the muzzle velocity of the projectile is increased. The internal space for the expansion of the powder charge increases as the square of the

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* *Vide* example, p. 19.

† *Vide* "Report on the Experimental Rifled M.L. Gun of 38 tons."

calibre, so that more work is done by the powder in the larger calibre; or, looking at it in another point of view, as the calibre increases so the base of the projectile is enlarged, and thus a greater surface is exposed to the action of the powder, and consequently the projectile is pushed forward with a greater aggregate pressure, and hence a higher muzzle velocity for a given length of bore results.

But there are other considerations which limit the calibre of guns. For when the calibre of the gun is increased,

(1) The total rending force, tending to burst the gun, increases (supposing the internal pressure per square inch the same) as the calibre.

(2) The resistance of the air to the motion of the projectile increases, and with projectiles of similar weight the velocity is lost more rapidly.

The first consideration operates most powerfully in the heaviest guns; the second in field and siege guns for direct fire.

In experiments with the 38-ton gun, the 12·5-in. calibre imparted a higher muzzle velocity to similar projectiles, when fired with equal charges, than the 12-in. calibre—which was one of the reasons why the 12·5-in. calibre was eventually adopted.

With field guns, on the contrary, the result of experiments has been such as to justify a diminution of calibre for the same weight of projectile; since the remaining velocity, at ordinary artillery ranges, has an important practical effect on the precision and decision of fire.

In order to test the effect of the variation of the calibre of a gun, when firing the same weight of projectile with the same charge, both as regards muzzle velocity and remaining velocity, at a given range, experiments have been made with the polygroove M.L. 12-pr. gun to determine the remaining velocity of the projectile at 1040 yds. from the muzzle, by means of Le Boulengé's chronograph, with the following results:—

Table showing the Result of varying the Calibre of Field Guns.

Weight of projectile.	Weight of charge.	Calibre of gun.	Muzzle velocity.	Remaining velocity at 1040 yds.	Loss of velocity.
lbs.	lbs.	ins.	f.s.	f.s.	f.s.
12	3	3·0	1690	1208	482
12	3	3·2	1718	1144	574

This shows that although the muzzle velocity is slightly greater with the larger (3·2-in.) calibre, yet the remaining velocity at 1040 yds. is considerably greater with the smaller (3·0-in.) calibre.

Diameter of chamber, and cubic space occupied by the charge.

In the more recently constructed M.L. guns the diameter of the powder-chamber has been made larger than the diameter of the bore (or the calibre of the gun); partly to obviate the disadvantage above alluded to, of increasing the diameter of the projectile, and partly to obtain a higher muzzle velocity, with more uniform results.

The muzzle velocity, under certain conditions,* is slightly increased by enlarging the powder-chamber; as the cartridge becomes shorter, and more space is allowed for the expansion of the powder charge. The charge is probably exploded more uniformly and rapidly in a chambered gun; but the disadvantages of too rapid an explosion may be modified by the use of a slow-burning powder, and also by allowing sufficient *air space* behind the projectile, by means of which the extreme pressure of the powder gas is relieved at the critical moment. By chambering the gun, a larger charge of powder may be profitably burnt, with this advantage—that a higher muzzle velocity is obtained with the same maximum pressure per square inch in the chamber, than with a smaller charge in an unchambered gun.

Experiments with chambered guns have been carried out in the Royal Gun Factories, Woolwich, 1873, by Major Maitland, R.A. A 10-in. M.L. gun was chambered to a diameter of 12 ins. for a length of 19 ins. It was fired with a charge of 85 lbs. P., which gave a less maximum pressure than the unchambered gun with the service charge (70 lbs. P.), while the muzzle velocity was increased by 75 f.s.†

But when large charges are rammed up tight, it is found that the pressure in the *powder-chamber* increases, as well as the muzzle velocity —the former sometimes to a dangerous extent. The object of the artillerist is to get the *greatest amount of work out of a gun with safety*, quite irrespective of the means employed. To obtain a high muzzle velocity with a low maximum pressure, it is necessary to maintain that pressure as much as possible while the projectile is moving along the bore. This may be done by *increasing* the ordinary charge of powder (when rammed up tight), and *giving air-space*, so as to allow for the expansion of the powder gas at the first instant of explosion; thus tending to keep the maximum pressure in the powder-chamber within safe limits.

Air-space is usually given by lengthening the cartridge, and then ramming home the projectile, so as just to touch it; but the principle of the employment of an air-space depends on the ratio of the bulk of the charge to the number of cubic inches of space in rear of the projectile. It is usually measured by the number of cubic inches occupied by each pound of powder in the charge, when the projectile is in its proper position for firing.

The amount of cubic space thus occupied by the cartridge has a very marked influence on the pressure in the gun and the muzzle velocity of the projectile. The effect of increasing the cubic space for a given charge of powder is to reduce both the muzzle velocity and also the maximum pressure in the powder-chamber; so that a much larger charge of powder may be fired with safety when a certain amount of air-space is allowed. This has led to the employment of much larger charges in the heaviest guns than were originally contemplated; the maximum pressure being still kept within safe limits, while a considerable increase in muzzle velocity has been obtained.

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* With small charges, chambering reduces velocity; with medium charges, slightly increases velocity; with large charges, greatly increases velocity; but in all cases reduces pressure.

† *Vide* "Treatise on the Construction and Manufacture of Ordnance in the British Service."

Some experiments carried out by the Committee on Explosives with the 38-ton gun, with cubical powder, will serve to illustrate these principles:—

Table showing Experiments with 38*-ton Gun, allowing air-space.*
(Cubical powder, 1·5 ins.)

Number of round.	Charge.	Shot.	Muzzle velocity.	E, Energy.	P, Mean press. in powder-chamb. per sq. in.	$\frac{E}{P}$	Length of cartridge.	Cubic ins. per pound of powder.
	lbs.	lbs.	f.s.	ft. tons.	tons.		ins.	
1	130*	800	1451	11655	24·5	476·9	28·00	24·6
2	130*	800	1391	10734	19·3	556·1	33·22	30·0
3	130*	800	1541	13176	22·4	587·3	45·37	30·0
4	130†	800	1544	13225	20·8	634·0	36·25	30·0

* Gun unchambered. † Gun chambered.

When cubical powder is rammed up hard, each pound of powder occupies about 24·6 cub. ins.: any additional cubic space occupied shows that a certain amount of air-space has been given. The most suitable amount of air-space depends upon the system of rifling adopted, the nature of the powder, the weight of charge and projectile, and must be experimentally determined. With Woolwich guns and cubical pebble, the most suitable amount of air-space has been found to vary, under present conditions, from 30 to 34 cub. ins. per pound of powder.*

The effect of allowing air-space is well shown by comparing rounds (1) and (2) in the above table. In round (1) the charge is rammed up hard; in round (2) air-space is given, allowing 30 cub. ins. for each pound of powder. The result is that both energy and pressure are reduced, but the *ratio* of energy to pressure is increased—which is important.

The column $\frac{E}{P}$ represents the ratio of energy of projectile to the mean pressure per square inch in the powder-chamber; and the greater this ratio is (while the pressure, P, is kept within safe limits), the more effective is the gun.

Round (3) shows how, under similar conditions of cubic space per pound of powder, the charge may be increased, and consequently the energy of the projectile, while the mean pressure in the powder-chamber is kept within safe limits. The cartridge is rather long—which is a disadvantage, as it has been found that there is a danger in firing very long cartridges (even when air-space is given) of setting up *wave action*, which increases the local pressure in the powder-chamber abnormally. It becomes, therefore, of considerable importance, when very large charges are fired, to chamber the gun—*i.e.*, to increase the diameter of the powder-chamber, and so to reduce the length of the cartridge and the pressure in the chamber.

Round (4) shows the advantage of enlarging the chamber. The weight of charge and projectile are the same as in round (3), but the

* It is not improbable, under certain conditions, that the amount of air-space may yet be still more increased.

cartridge is shorter, the energy slightly greater, the pressure in the powder-chamber less, and—what is of the most importance—the ratio of energy to pressure, $\frac{E}{P}$, is the greatest of all the rounds fired.

When the powder-chamber is enlarged, it must be remembered that the strain on the chamber at any point, or the tendency to burst at that point, depends not upon the pressure per square inch at that point only, but upon the *product* of the pressure per square inch at that point into the number of inches in the diameter; or, in other words, if the pressure per square inch in the chamber remains the same when the chamber is enlarged, the rending strain on the chamber increases as the diameter.

Windage. The disadvantages of windage have already been pointed out, and the steps which have been taken to remedy these defects by the adoption of gas-checks in the heavy guns. (Page 6.)

The escape of gas by windage not only causes erosion of the bore, but also diminishes the mean pressure on the base of the projectile, and consequently reduces its muzzle velocity. With muzzle-loading guns windage is necessary, to allow of facility of loading; but in all these guns the smallest amount of windage is given consistent with ease and rapidity of loading. In service guns there is no difficulty in sealing the windage, by fixing on a gas-check to the base of the projectile, so that it may be loaded easily, and that, on firing, the shock of discharge may expand it, and seal up the windage. This has been successfully accomplished with the heavy guns, resulting in *increased muzzle velocity*, and consequently greater range and accuracy, and greater power of penetration. This method of sealing the windage will also, in all probability, be applied to muzzle-loading field and siege guns of future construction.

The Committee on Explosives state "that the use of copper gas-checks in the 38-ton gun, when firing 130-lb. charges of 1·5-in. cubical powder, and 800-lb. projectiles, gives an increase of muzzle velocity of about 38 ft. secs. over that obtained with a projectile of the same weight with service *windage*, fired with the same charge—a gain equivalent to an increase in the length of the bore of the gun of about three calibres. This increase is evidently due to the sealing of the windage by the gas-check, whereby the pressure in the powder-chamber is increased slightly, while there is also an increased pressure on the base of the shot."*

Experiments with 35 and 38-ton guns. A comparison of the results obtained with the 35-ton gun of 12-in. calibre and the 38-ton gun of 12·5-in. calibre, illustrates the above principles. The 38-ton gun was 3 ft. longer in the bore than the experimental 35-ton gun; but in all other respects they were identical, except in calibre. The original charge in the 35-ton gun was 110 lbs. P.

The increase in muzzle velocity due to the various points of difference between the experimental 35-ton gun and the present 38-ton gun, is *approximately*† as follows:—

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* *Vide* "Report on the Experimental Rifled M.L. Gun of 38 tons."

† The weight of the projectile was 700 lbs. with the 12-in. calibre, and was increased to 800 lbs. with the 12·5-in. calibre.

		f.s.
(1)	Increase in velocity due to lengthening the bore 3 ft.* ... =	54
(2)	Increase in velocity due to increase in calibre from 12 ins. to 12·5 ins. ..... =	30
(3)	Increase in velocity due to use of gas-check ..... =	38
(4)	Increase in velocity due to adoption of cubical pebble, and increasing charge from 110 lbs. P. to 130 lbs. P^2. ..... =	35
(5)	Increase in velocity due to still greater increase of charge from 130 lbs. to 180 lbs. $P.^2$, made practicable by allowing air-space and chambering ..... =	150

Position of the vent.

The position of the vent, or the point of ignition of the cartridge, makes a difference (especially when R.L.G. powder is employed) in the rate of explosion of the charge, and influences the muzzle velocity of the projectile, and the pressure in the powder-chamber.

All the heavy M.L. guns are vented on the same principle—*i.e.*, the position of the vent (where the cartridge is ignited) is fixed at a distance of $\frac{4}{10}^{ths}$ of the length of the battering charge from the bottom of the bore. This position of the vent was found to develop the greatest muzzle velocity, when firing large charges of R.L.G. powder. It was found with cubical pebble powder that there was no difference in muzzle velocity, whether the cartridge was lighted at the rear or in the service position, but that there was a considerable increase in the chamber pressure near the base of the projectile when lighted at the rear. Arrangements have therefore been made in the 80-ton gun to light the cartridge at the service position (although using an axial vent), in order to keep the chamber pressure as low as possible.

Field and siege guns are *rear* vented—(1) so as to ensure the ignition of the cartridge, when firing reduced charges, (2) so as to expel the *débris* of the cartridge of a preceding round, when firing service charges, in order to avoid an accident in re-loading. But if these guns were vented near the centre of the cartridge, as in the heavy guns, there would be a gain in muzzle velocity. Experiments have been carried on with rear and forward vents, with the 16-pr., 25-pr., and 40-pr. guns.

Table showing the Effect of altering the Position of the Vent on Muzzle Velocity.

Gun.	Charge, R.L.G.	Muzzle velocity with rear vent.	Muzzle velocity with forward vent.	Gain in M.V.
	lbs.	f.s.	f.s.	f.s.
16-pr.	3	1358	1397	39
25-pr.	5	1332	1430	98
40-pr.	8	1326	1470	144

In the Armstrong B.L. guns the point of ignition is in the axis of the piece; so that the cartridge is ignited at one end, and probably takes a longer time to explode than if ignited near the centre.

There are so many elements of variation with regard to the explosion of different charges of different powders fired from different guns, already pointed out, that sometimes apparently contradictory results are obtained; so that it may be concluded that probably the best position of vent in any gun is that one which offers the most convenience and safety in practice; and then, if necessary, the rate of explosion can

* That is, the bore was lengthened from 162 to 198 ins.

be modified by the nature of powder, form of chamber, amount of air-space, &c., found advisable under the circumstances.

There is a small loss of velocity due to the escape of gas through the vent. In the heaviest guns it becomes of importance to close the orifice of the vent, and so to prevent the wear of the bush caused by the action of the escaping gas.*

Twist of rifling.

The muzzle velocity of a projectile is affected, under similar conditions, by the amount of work absorbed in giving it rotation; but the work absorbed in giving rotation is so small† (about ·2 per cent. of the whole with the 16-pr. field gun), that practically there is very little difference in the muzzle velocity from this cause in service guns.

From experiments made with the 8-in. M.L. gun, rifled (1) with increasing twist—0 to 1 in 40 calibres, (2) with uniform twist—1 in 40 calibres, and fired under similar conditions of weight of projectile and charge of powder, it was found that in point of accuracy of fire the increasing twist had the advantage, but that the gun with uniform twist had a higher muzzle velocity and consequent length of range.‡

Table showing the Effect of the Employment of Uniform or Increasing Twist on Muzzle Velocity.

Spiral, 1 in 40 calibres.	Charge of powder, R.L.G.	Weight of projectile.	Muzzle velocity.
	lbs.	lbs.	f.s.
Increasing twist ..	30	179	1303·3
Uniform twist.....	30	179	1338·6

The Ordnance Select Committee stated "that the supposed advantage of the increasing spiral is, that the projectile having, at the moment of leaving its seat, only to move directly forward, gets away from the powder charge more readily than when—as in a gun with uniform spiral—it is forced simultaneously into rotatory motion. Somewhat more of the powder is therefore consumed before the shot moves in the latter than in the former case, and therefore the total force acting on the projectile during its passage through the bore is somewhat greater, and hence a somewhat greater muzzle velocity."

But this would hardly account for all the difference of velocity observed. The friction between the driving edges of the studs and the sides of the grooves in the gun is greater in the increasing than in the uniform twist for the same angle of spiral at the muzzle; for, on referring to the table at p. 16, giving the total pressure on studs (1) in uniform (2) in increasing twist, it will be seen that although the maximum pressure on the studs at any point in the bore is less in the increasing than in the uniform twist, yet the aggregate of the pressures is greater in the increasing twist, and consequently the friction resulting therefrom is greater in the increasing than in the uniform twist. This friction absorbs some of the work done by the charge of powder, and a lower muzzle velocity results. The small loss of muzzle velocity in consequence of the employment of increasing twist, is, in the opinion of practical artillerists, more than counterbalanced by its advantages in other respects.

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* *Vide* "Treatise on the Construction and Manufacture of Ordnance in the British Service," p. 55.

† *Vide* p. 15, note.

‡ *Vide* "Proceedings, Ordnance Select Committee," Vol. IV. p. 252.

CHAPTER V.

Resistance to the Motion of Projectiles in the Air.

Resistance of the Air.—Robins' Experiments.—Hutton's Experiments.—Hutton's Law.—Didion's Law.—Welter's Law.—Hélie's Law.—Bashforth's Experiments.—Report of Committee of Reference on Bashforth's Experiments.—Advantage of Bashforth's Method of Experimenting.—Object of the Experiments.—Bashforth's Conclusions.—Cubic Law retained for Convenience of Calculation.—Mayevski's Experiments.—Influence of the Form of the Projectile upon the Resistance.—Experiments to Determine the amount of Resistance of the Air to Different Forms of Heads of Projectiles.—Results of Experiments.—The Resistance of the Air to a Projectile in Motion.—The Retardation varies directly as the Resistance opposed to the Motion of the Projectile, and inversely as the Weight of the Projectile.—Equation of Motion connecting Distance and Velocity.—Approximate Calculation of Remaining Velocity with regard to Range.—The Value of the Coefficient K varies at Different Velocities.—Explanation and Use of the Table of Values of the Coefficient K.—Table connecting Distance and Velocity.—The Power which a Projectile in Motion has of maintaining its Velocity.—Equation of Motion connecting Time and Velocity.—Approximate Calculation of Remaining Velocity with regard to Time.—Distance and Velocity Table.—Time and Velocity Table.—Explanation of Tables I. and II.—Equation (I.), connecting Distance and Velocity.—Equation (II.), connecting Time and Velocity.—Examples in the Solution of Problems in Practical Gunnery by means of Tables I. and II.

Resistance of the air.

The resistance of the air to projectiles moving in it with high velocities has long been known to be very considerable, and many attempts have been made to find a *law* which would account for the results observed in practical gunnery.

Robins' experiments.

Robins' experiments in 1742 are the first recorded attempts to obtain the law of the resistance of the air to projectiles moving with high velocities. He fired spherical leaden bullets from a small-arm, and by means of the ballistic pendulum deduced the following general statements:—(1), that till the velocity of the projectile exceeds 1100 f.s., the resistance of the air may be taken to be as the *square of the velocity;* (2), that at the velocity of 1100 f.s. (nearly the same as that with which sound is propagated through the air), this law of the resistance is *altered;* (3), that if the velocity exceeds 1100 f.s., then the absolute quantity of that resistance in these greater velocities will be nearly three times as great as it should be by a comparison with the lower velocities.*

Hutton's experiments.

Hutton afterwards (1790) made experiments with guns firing large spherical iron projectiles, and an improved ballistic pendulum, and came to the conclusion—(1), that the law of the resistance of the air was in a *higher ratio* than the *square of the velocity*, even at very low velocities; (2), that this ratio gradually increased with velocities up to 1600 f.s., where it is at the greatest, amounting in that maximum state to $2\frac{1}{10}$ times (instead of 3 times, as Robins asserted,) the quantity

* *Vide* Robins' "Tracts on Gunnery," by Hutton, p. 181.

resulting from the ratio of the square of the velocity; (3) that for velocities greater than 1600 f.s. this ratio gradually *decreased.*

Hutton's law.

Hutton's law of the resistance of the air, deduced from his experiments, consisted of two terms—one varying as the velocity, the other as the square of the velocity; *i.e.*, if R denote the resistance of the air, and v the velocity of the projectile,

$$R \propto av + bv^2;$$

where a and b are constants determined experimentally.

Didion's law.

General Didion, of the French army, proposed another law of resistance, which, from experiments carried on at Metz in 1840, he considered to be nearer the truth. Didion's law of resistance also consisted of two terms—one varying as the square, the other as the cube of the velocity, or

$$R \propto \cdot 027\ (v^2 + \cdot 0023v^3).*$$

Welter's law.

This formula not having proved entirely satisfactory, experiments were again carried on at Metz, in 1857, by Captain Welter, Professor at the School of Practical Artillery and Engineering, but this time with improved ballistic instruments. The old ballistic pendulum was discarded, and the velocities of spherical projectiles were taken by means of an *electro-ballistic* pendulum.† Captain Welter came to the conclusion that the resistance of the air to *spherical* projectiles moving at high velocities was simply proportional to the *cube of the velocity;*

$$\text{or } R \propto v^3.$$

Hélie's law.

Further experiments were carried on in 1860–1 at Gâvre with ogival-headed *elongated* projectiles, under the direction of M. Hélie (Professor in the French School of Naval Gunnery), from which he came to the conclusion, in his work "Traité de Balistique Experimentale," published in 1865, that the results of these experiments "authorised him to consider that the resistance of the air was proportional to the cube of the velocity, at least as long as the axis of the projectile did not deviate much from the tangent to the trajectory described by its centre of gravity;"‡ but it was not until Professor Bashforth carried out a series of systematic experiments with spherical and ogival-headed projectiles by means of his Clock Chronograph (1865 to 1870) that any *accurate* knowledge of the subject was obtained. A Committee of Reference appointed by the War Office, consisting of the Astronomer Royal, Professor Adams (Cambridge Observatory), Professor Stokes, of Cambridge, and Captain Andrew Noble, F.R.S., of Elswick, reported thus on these experiments:—

Bashforth's experiments.

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* *Vide* "Traité de Balistique Experimentale," par Helié, p. 168.

† An electro-ballistic pendulum is an instrument by which the exact instant of the passage of a projectile through a screen is registered by *electricity*, and the time of passing from one screen to another by means of a pendulum falling freely. These instruments have since been superseded by the more accurate instruments of Le Boulengé, Bashforth, and Watkin. (*Vide* Appendix I.)

‡ *Vide* "Traité de Balistique Experimentale," p. 415.

"The results showed that for moderate variations of velocity the resistance may be taken to vary as the *cube of the velocity;* yet for considerable variations of velocity this law no longer holds good; and that if for convenience the resistance be still represented by an expression of the form cv^3, the coefficient c must be taken to vary continuously with the velocity. The experiments showed the values of the coefficient c of resistance corresponding to values of the velocity ranging from 850 to 1600 ft. per second. The coefficient is found to attain a maximum value for a velocity of about 1200 f.s. Professor Hélie's value of the coefficient of resistance is found to be true only for velocities in the neighbourhood of 950 f.s. The (cubic) law was stated by Professor Hélie in his work published in 1865, and it appears to have been in use for some years before that time in the School of Artillery at Metz; but Professor Bashforth, without being aware of this, independently deduced the same law from his own experiments, so far at least as the law can be considered true—viz., *for a limited range of velocity.* Professor Bashforth, by his valuable experimental determination of the coefficient of resistance corresponding to different velocities, has furnished the data which are *absolutely necessary* in order to make the law applicable in practice."*

Report of Committee of Reference on Bashforth's experiments.

Bashforth's method of experimenting consisted in observing the time of the passage of the *same* projectile through a series of successive equal distances.† Previous experiments had measured only two intervals of time at each round, and in order to obtain the coefficient of resistance for different velocities, several rounds were fired with varying charges to get the required velocities; whereas Bashforth, by arranging 10 screens at intervals of 50 yds. apart, in *one round* determined the coefficient of resistance for a considerable variation of velocity—the velocity being gradually reduced in the distance of 450 yds. by the resistance of the air.‡

Advantage of Bashforth's method of experimenting.

The Committee of Reference reported with regard to this point that "there is a great advantage in being able to measure *several* consecutive intervals of time by the same instrument; since in this way each experiment supplies the means of testing the *accuracy* of the results, which are wholly wanting when only two intervals of time are measured, and that by two different instruments."

The object of these experiments was to determine whether the resistance of the air varied as the *cube of the velocity* for all velocities between 900 f.s. and 1700 f.s., and whether it varied also as the *square of the diameter* of the projectile.

Object of the experiments.

The results showed that the resistance of the air for the same velocity and the same *form* of projectile, whether spherical or elongated, varies exactly as the *square of the diameter* of the projectile; also that the resistance of the air varies as the *cube of the velocity only for velocities between 1100 f.s. and 1350 f.s.*

Bashforth's conclusions.

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* *Vide* Report of the Committee of Reference (p. 160 of "Reports on Experiments made with the Bashforth Chronograph.")

† *Vide* Bashforth's "Motion of Projectiles," p. 29.

‡ For description of Bashforth's Clock Chronograph, see Appendix I.

The cubic law is not nearly correct for velocities between 900 f.s. and 1100 f.s.: the resistance of the air between these velocities varies approximately as the *sixth* power of the velocity.

The cubic law, again, is not correct for velocities above 1350 f.s.: the resistance of the air above this velocity varies approximately as the *square* of the velocity.

Bashforth's conclusions are, then, that

between the velocities of 900 and 1100 f.s.... $R \propto v^6$, approximately,
" " 1100 and 1350 f.s.... $R \propto v^3$, "
above 1350 f.s. ..... $R \propto v^2$. "

Cubic law retained for convenience of calculation.

Since the cubic law of resistance offers the greatest simplicity of calculation, Bashforth retained it for convenience as the basis for the calculation of remaining velocities, and represented the resistance of the air by an expression of the form cv^3; where the coefficient c is *variable,* depending upon the *form* of the projectile and upon the *velocity* with which it moves.

Mayevski's experiments.

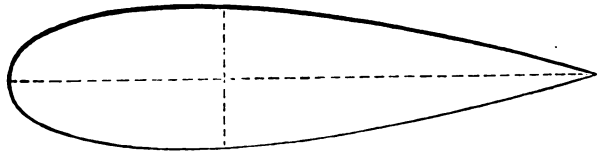
Experiments on the resistance of the air to elongated projectiles in motion were made at St. Petersburg in 1868 and 1869, under the superintendence of General Mayevski, by means of two Boulengé chronographs. Mayevski supplemented his experiments by using the coefficient of resistance deduced by Bashforth, and adopted a method of calculation* of the trajectory of a projectile on the suppositions (which he considered borne out by the experiments) that

between the velocities of 1670 and 1180 f.s.... $R \propto v^2$, approximately,
" " 1180 and 918 f.s.... $R \propto v^6$, "
below 918 f.s. ..... $R \propto v^2 + v^4$, "

The calculations on these suppositions are not so simple as those obtained by using Bashforth's method of formulating the results of his experiments.

Influence of the form of the projectile upon the resistance.

The resistance experienced by a projectile in motion through the air is influenced by its *form or shape*—more especially by the shape of its *fore* part, which meets the air, although the shape of its *hinder* part has some influence on the resistance. The form which would theoretically encounter the least resistance in passing through the air is given by Piobert, in his "Cours d'Artillerie," p. 15.



* *Vide* "Traité de Balistique Exterieure," par N. Mayevski. Paris, 1872.

Its length is five times its greatest diameter, and its largest section is placed at $\frac{2}{3}^{\text{ths}}$ of its length from the base.*

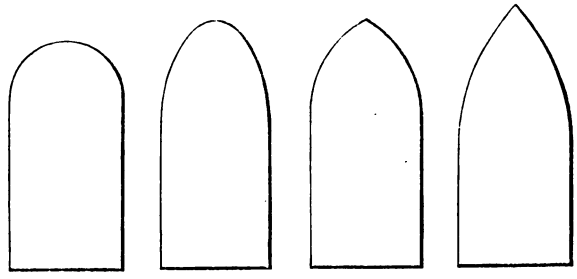
With service projectiles the base is generally flat, but slightly rounded at the edges for convenience in loading.

The total resistance to a projectile moving through the air is due (1) to the fore part of the projectile forcing its way through, and so displacing the particles of the air which impede its motion; (2) to the hinder part being dragged back by suction, owing to a formation of a partial vacuum in its rear.

Experiments to determine the amount of resistance of the air to different forms of projectiles.

Experiments were made by Professor Bashforth with the Clock Chronograph to ascertain the amount of the resistance of the air to different forms of heads of projectiles, the form of the hinder part being flat in all.

Four different forms of head were experimented on—viz., (1) hemispherical head, (2) hemispheroidal head, (3) ogival head of 1 diameter, and (4) ogival head of 2 diameters. The object of the experiment was to determine which of these forms offered the least resistance to the motion of the projectile.



1. 2. 3. 4.

Results of experiments.

The results obtained under similar conditions of velocity were as follows:—

Form of head.	Coefficient of resistance $K = (1000)^3 c \times \frac{w}{d^2}$.
Hemispherical	132·9
Hemispheroidal	104·3
Ogival-head (1 diam.)	109·7
Ogival-head (2 diams.)	104·2

The coefficient c is put into the form $K = (1000)^3 c \times \frac{w}{d^2}$ for convenience of calculation; d being the diameter of the projectile in inches, and w its weight in lbs.: so that

$$c = \frac{d^2}{w} \frac{K}{(1000)^3}.$$

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* *Vide* Owen's "Modern Artillery," p. 215.

It was concluded that the resistance of the air to the hemispherical head was the greatest, and to the ogival head of 2 diameters the least, of the forms experimented on.

The resistance of the air to the hemispheroidal and ogival heads differs so little that there is practically not much to choose between them. Professor Bashforth states that "the slight variations in the resistances to the three latter forms lead to the conclusion that the amount of resistance offered by the air to the motion of elongated projectiles is little affected by the more or less pointed apex, but depends chiefly upon the form of head near its junction with the cylindrical body of the projectile. In this neighbourhood the forms of the hemispheroidal head and the ogival head struck with a radius of 2 diameters are the same, and the resistances are little different."

The resistance of the air to a projectile in motion.

THE RESISTANCE WHICH AN ELONGATED PROJECTILE IN MOTION MEETS WITH IN PASSING THROUGH THE AIR DEPENDS

(1) On the velocity of the projectile.
(2) On its sectional area.
(3) On the form of head.

For velocities between 1100 f.s. and 1350 f.s. the resistance varies as the *cube of the velocity;* so that if $v =$ the velocity of projectile, $d =$ its diameter, K' a coefficient depending on the form of head, the resistance opposed to its motion,

$$\text{or } R, \text{ varies as } v^3 d^2 K'.$$

The retardation varies directly as the resistance opposed to the motion of the projectile, and inversely as the weight of the projectile.

For projectiles with similar forms of heads K' is constant; so that within the limits of velocity mentioned above, and with ordinary service projectiles,

$$R \propto K' d^2 v^3 = \frac{K' d^2 v^3}{g}, \text{ suppose.}$$

The retardation, or the rate of loss of velocity, of the projectile owing to this resistance, varies *inversely as the weight of the projectile;* so that if w be the weight of the projectile, it follows that

$$\text{the retardation} = \frac{gR}{w} = K' v^3 \frac{d^2}{w}.$$

For convenience of calculation, suppose $K' = \frac{K}{(1000)^3}$; then the above relations become

$$\text{resistance } (R) = K \cdot \frac{d^2}{g} \cdot \left(\frac{v}{1000}\right)^3, \quad \ldots\ldots(1)$$

Equation of motion connecting distance and velocity.

$$\text{and retardation} = K \frac{d^2}{w} \left(\frac{v}{1000}\right)^3, \quad \ldots\ldots(2)$$

$$\text{or } v \frac{dv}{ds} = -\frac{d^2}{w} \frac{K v^3}{(1000)^3},$$

supposing the projectile to move in a straight line, unaffected by gravity;

$$\text{then } \frac{ds}{dv} = -\frac{(1000)^3}{Kv^2} \cdot \frac{w}{d^2},$$

$$\text{or } s = \frac{w}{d^2}\int \frac{(1000)^3 dv}{Kv^2}; \quad \ldots\ldots(3)$$

the limits of this integral being the velocities at the beginning and end of the distance, s.

Approximate calculation of remaining velocity with regard to range.

For velocities between 1100 f.s. and 1350 f.s., where the cubic law holds, the coefficient K may be given a constant value for the same form of projectile. Between these velocities for service ogival-headed projectiles

$$K = 108{\cdot}5.$$

When K is constant, equation (3) becomes

$$s = \frac{w}{d^2}\frac{(1000)^3}{K}\int \frac{dv}{v^2},$$

which may be integrated thus:

$$\frac{K}{(1000)^3}\frac{d^2}{w}s = \frac{1}{v} - \frac{1}{V},$$

$$\text{or } \frac{d^2}{w}s = \left(\frac{1}{v} - \frac{1}{V}\right)\frac{(1000)^3}{K}; \quad \ldots\ldots(4)$$

where V is the muzzle velocity, and v the remaining velocity, at a given distance, s, from the muzzle.

Or, transposing,

$$v = \frac{V}{1 + \frac{K}{(1000)^3}\frac{d^2}{w}Vs}; \quad \ldots\ldots(5)$$

which gives the remaining velocity at a given distance, s, from the muzzle.

For velocities where the cubic law holds, and the projectile moves approximately in a straight line, the remaining velocity at a given *distance* may be fairly accurately determined by this formula by substituting the mean value of K given above.

For example: suppose it were required to find the remaining velocity of the 10-in. M.L. gun of 18 tons at 1000 ft.*

$$V = 1364 \text{ f.s.}, \quad \frac{d^2}{w} = {\cdot}24, \quad K = 108{\cdot}5, \text{ and } s = 1000.$$

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* *Vide* Table, "Comparison of M.L. Guns," p. 20.

These values substituted in equation (5) will give (v) the remaining velocity at 1000 ft.

$$\text{Thus } v = \frac{1364}{1 + \frac{108{\cdot}5}{(1000)^3} \times {\cdot}24 \times 1364 \times 1000}$$

$$= 1317 \text{ f.s.}$$

By the use of Bashforth's Distance and Velocity Table (*vide* p. 55) calculations of this kind are much simplified.

The value of the coefficient K varies at different velocities.

The value of K, however, cannot be taken as constant for large variations in velocity; so that it becomes necessary to tabulate the value of K for all practical velocities.

Up to the present time the value of this coefficient has been accurately determined for velocities varying from 1700 f.s. to 900 f.s. for ogival-headed projectiles. Experiments are now in contemplation to determine it for the lower velocities used in curved or indirect fire.

*Table showing the Values of K corresponding to Velocities at Intervals of 10 ft. per second. Elongated shot, ogival head.**

Velocity.	Value of K.	Velocity.	Value of K.	Velocity.	Value of K.	Velocity.	Value of K.
f.s.		f.s.		f.s.		f.s.	
900	64·4	1110	106·6	1310	107·7	1510	96·4
910	64·8	1120	107·1	1320	107·4	1520	95·5
920	65·3	1130	107·5	1330	107·1	1530	94·7
930	65·9	1140	107·9	1340	106·8	1540	93·8
940	66·6	1150	108·2	1350	106·4	1550	93·0
950	67·4	1160	108·5	1360	106·0	1560	92·2
960	68·4	1170	108·7	1370	105·6	1570	91·4
970	69·6	1180	108·9	1380	105·1	1580	90·6
980	71·0	1190	108·9	1390	104·6	1590	89·8
990	72·8	1200	108·9	1400	104·0	1600	89·0
1000	75·0	1210	108·9	1410	103·4	1610	88·2
1010	77·5	1220	108·9	1420	102·8	1620	87·4
1020	80·4	1230	108·8	1430	102·2	1630	86·7
1030	83·9	1240	108·8	1440	101·6	1640	86·0
1040	88·2	1250	108·7	1450	100·9	1650	85·4
1050	92·8	1260	108·6	1460	100·2	1660	85·0
1060	97·2	1270	108·5	1470	99·4	1670	84·6
1070	100·8	1280	108·3	1480	98·7	1680	84·3
1080	103·4	1290	108·1	1490	97·9	1690	84·1
1090	105·1	1300	107·9	1500	97·2	1700	83·9
1100	106·0						

N.B.—In the above table a cubic foot of dry air is supposed to weigh 530·6 grs., which corresponds to the barometer at 30·0 ins. and thermometer at 65·5° F. The value of K varies as the density of the air, but it is calculated at a standard density on the above supposition.

Explanation and use of the Table of Values of the Coefficient K.

Suppose the value of the coefficient K be required for a velocity of 1600 f.s.: opposite to 1600 in the velocity column, $K = 89{\cdot}0$.

EXAMPLE.—What is the resistance of the air in lbs. to an ogival-headed projectile of 16 ins. in diameter, moving with a velocity of 1600 f.s.?

* *Vide* p. 152, "Report on Experiments made with the Bashforth Chronograph to determine the Resistance of the Air to the Motion of Projectiles."

From equation (1),

$$R = K\frac{d^2}{g}\cdot\left(\frac{v}{1000}\right)^3 = \frac{89\times(16)^2}{32{\cdot}19}\left(\frac{1600}{1000}\right)^3 = 2899 \text{ lbs.}$$

By substituting the mean value of K in the above table corresponding to the mean velocity $\frac{V+v}{2}$ in equation (5), the remaining velocity, v, may be approximated to; but in order to get accurate results it would be necessary to change the value of K continually, to correspond to the continual change of velocity owing to the resistance of the air, which would be a laborious calculation.

Table connecting distance and velocity.

In order to obviate this, Mr. Bashforth has tabulated the values of $\frac{d^2}{w}s = \left(\frac{1}{v}-\frac{1}{V}\right)\frac{(1000)^3}{K}$ for all values of K corresponding to velocities from 1700 f.s. to 500 f.s. In the absence of exact experiments for velocities below 900 f.s., it has been assumed that the coefficient K remains constant and equal to its value at 900 f.s., and that the cubic law holds good below 900 f.s. (*Vide* Table I., Distance and Velocity, p. 55.)

The power which a projectile in motion has of maintaining its velocity.

It has been stated that the rate at which a projectile loses velocity owing to the resistance of the air equals

$$K\frac{d^2}{w}\cdot\left(\frac{v}{1000}\right)^3;$$

[*vide* equation (2)]; but as K is a coefficient depending only on the form and velocity of the projectile, it follows for similarly shaped projectiles fired from guns of *different calibres* and moving with the same alterations of velocity, that

$$\text{the rate of loss of velocity varies simply as } \frac{d^2}{w};$$

or, inverting and putting it into other words,

$$\text{the power which a projectile has of } \textit{maintaining its velocity}$$

$$\text{varies as } \frac{w}{d^2} \text{ or } \frac{\text{weight of projectile}}{\text{square of its diameter}};$$

or the power of a projectile to maintain its velocity varies *directly as its weight, and inversely as the square of its diameter.*

With similarly shaped elongated projectiles the weight varies nearly as d^2l, where l is the length of the projectile; consequently

$$\text{power of the projectile} \propto \frac{w}{d^2} \propto \frac{d^2l}{d^2} \propto l;$$

or the power of the projectile varies as its length.

Thus the longer the projectile (*cæteris paribus*) the harder will it hit at any given range, and the greater will be its absolute range for any

given muzzle velocity; but practical considerations limit the length of a projectile to about 3 calibres.

If elongated projectiles of similar shape were made the *same length in calibres*, it follows from what has been stated above that

the power of the projectile varies as the calibre of the gun.

Equation of motion connecting time and velocity.

IT NOW BECOMES NECESSARY TO INVESTIGATE THE CONDITIONS OF VELOCITY WITH RESPECT TO *time*.

From equation (2),

$$\text{retardation} = \frac{dv}{dt} = -\frac{d^2}{w} \cdot \frac{Kv^3}{(1000)^3},$$

supposing the projectile to move in a straight line unaffected by gravity; or, inverting and integrating,

$$t = \frac{w}{d^2} \int \frac{(1000)^3 dv}{Kv^3}; \quad \ldots\ldots\ldots\

velocities where the cubic law does not hold and the value of K changes rapidly, Bashforth has calculated equation (3), $\frac{d^2}{w}s = \int_{500}^{1700} \frac{(1000)^3 dv}{Kv^3}$, (the value of K continually changing, so as to have the proper value corresponding to the velocity), and tabulated the results in what may be called a *Distance and Velocity* Table (*vide* Table I., p. 55); also the value of equation (6), $\frac{d^2}{w}t = \int_{500}^{1700} \frac{(1000)^3 dv}{Kv^3}$, in what may be called a *Time and Velocity* Table (*vide* Table II., p. 57). Distance and Velocity Table. Time and Velocity Table.

Table I. connects distance and velocity, and is denoted by S_v; Table II. connects time and velocity, and is denoted by T_v.

The units employed are distance in feet, time in seconds, and velocity in feet seconds.

In Tables I. and II. the left hand vertical column (v) denotes velocities from 500 f.s. to 1700 f.s., at intervals of 10 f.s., and by means of the top horizontal column these velocities are subdivided into intervals of 1 f.s. Explanation of Tables I. and II.

The figures in the vertical columns under 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (Table I.), express the *distance in feet* corresponding to the velocities, and may be denoted by the abbreviation S_v.

Similarly, the figures in the vertical columns under 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (Table II.), express the *time in seconds* corresponding to the velocities, and may be denoted by the abbreviation T_v.

For example: in Table I., suppose it required to find the distance corresponding to the velocity 601 f.s.—*i.e.*, $S_v = S_{601} = 14384{\cdot}8$ ft.; or, conversely, if $S_v = 14384{\cdot}8$ ft., $v = 601$ f.s.

Also, in Table II., suppose it required to find the time corresponding to the velocity 705 f.s.—*i.e.*, $T_v = T_{705} = 11{\cdot}0732$ secs.; or, conversely, if $T_v = 11{\cdot}0732$ secs., $v = 705$ f.s.

Table I. is obtained by calculating the integral in the left-hand member of the equation Equation (I.), connecting distance and velocity.

$$\frac{d^2}{w} S = \int \frac{(1000)^3 dv}{Kv^2};$$

and when the integral is taken between the limits of velocity v and V, it may be expressed in the notation adopted in Table I. thus:

$$\frac{d^2}{w} S = \int_v^V \frac{(1000)^3 dv}{Kv^2} = S_v - S_V. \quad \text{.....(I.)}$$

In the same way, Table II. is obtained by calculating the integral in the right-hand member of the equation Equation (II.), connecting time and velocity.

$$\frac{d^2}{w} T = \int \frac{(1000)^3 dv}{Kv^3};$$

and when the integral is taken between the limits of velocity v and V, it may be expressed in the notation adopted in Table II. thus:

$$\frac{d^2}{w}T = \int_v^V \frac{(1000)^3 dv}{Kv^3} = T_v - T_V. \quad \text{.....(II.)}$$

In order to make use of Tables I. and II. in the solution of problems in practical gunnery, it must be remembered that they apply *rigidly* only to elongated projectiles with ogival-shaped heads, and under conditions of atmosphere mentioned in the note on the Table of Values of K, and that

V = the velocity of the projectile at the beginning of the distance or time under consideration,
v = the velocity at the end of the distance or time under consideration,
S = the distance or range in *feet*,
T = the time in *seconds*,
d = diameter of projectile in *inches*,
w = weight of projectile in *lbs.*

Examples in the solution of problems in practical gunnery by means of Tables I. and II.

EXAMPLE (1).—In what range would the velocity of a projectile fired from the 9-in. M.L. gun be reduced from 1420 f.s. to 1240 f.s.?

For the 9-in. shell, $d = 8{\cdot}92$ ins., $w = 250$ lbs.; also $V = 1420$, $v = 1240$.

Substituting in equation (I.),

$$\frac{d^2}{w}S = S_v - S_V;$$

$$\text{or } \frac{(8{\cdot}92)^2}{250}S = S_{1240} - S_{1420} = 2208{\cdot}6 - 1251{\cdot}1, \text{ by Table I.};$$

$$\text{or } {\cdot}3183S = 957{\cdot}5 \text{ ft.};$$

$$\text{so that } S = 3000 \text{ ft. nearly.}$$

i.e., the velocity would be reduced from 1420 f.s. to 1240 f.s. in about 3000 ft.

EXAMPLE (2).—In what *time* would the velocity of the same projectile be reduced from 1420 f.s. to 1240 f.s.?

Substituting in equation (II.),

$$\frac{d^2}{w}T = T_v - T_V;$$

$$\text{or } \frac{(8{\cdot}92)^2}{250}T = T_{1240} - T_{1420} = 1{\cdot}5285 - {\cdot}8057;$$

$$\text{or } {\cdot}3183T = {\cdot}7228 \text{ sec.};$$

$$\text{so that } T = 2{\cdot}27 \text{ secs.}$$

i.e., the velocity would be reduced from 1420 f.s. to 1240 f.s. in about 2·27 secs.

EXAMPLE (3).—Find the remaining velocity of the 12·5-in. M.L. gun at 3000 ft.

In this case, $w = 800$ lbs., $d = 12{\cdot}42$ ins., muzzle velocity $(V) = 1420$ f.s., $S = 3000$ ft.

Substituting in equation (I.),

$$\frac{d^2}{w} S = S_v - S_V;$$

$$\text{or } \frac{(12{\cdot}42)^2}{800} \times 3000 = S_v - S_{1420}.$$

Transposing,

$$S_v = \frac{(12{\cdot}42)^2}{8} \times 30 + S_{1420}$$

$$= 578{\cdot}4 + 1251{\cdot}1 \text{ (since } S_{1420} = 1251{\cdot}1 \text{ by Table I.)};$$

$$\text{or } S_v = 1829{\cdot}5 \text{ ft.};$$

then from Table I. $v = 1306{\cdot}6$ f.s.

i.e., the remaining velocity at the distance of 3000 ft. from the muzzle is 1306·6 f.s.

EXAMPLE (4).—A shell fired from the 64-pr. M.L. gun with a charge of 10 lbs. of powder was observed to strike the crest of a parapet in 3 secs.; the muzzle velocity is known to be 1383 f.s. Find the striking velocity and the range.

In this case, $w = 64$ lbs., $d = 6{\cdot}22$ ins., $V = 1383$ f.s., $T = 3$ secs.

Substituting in equation (II.),

$$\frac{d^2}{w} T = T_v - T_V;$$

$$\text{or } \frac{(6{\cdot}22)^2}{64} \times 3 = T_v - T_{1383};$$

$$\text{or } T_v = 1{\cdot}814 + {\cdot}9351 \text{ (since } T_{1383} = {\cdot}9351 \text{ sec. by Table II.)};$$

$$= 2{\cdot}7491 \text{ secs.};$$

then from Table II. $v = 1049$ f.s.

i.e., the striking velocity at the end of 3 secs. from firing = 1049 f.s.

Next, to find the range :—

Substituting in equation (I.),

$$\frac{d^3}{w} S = S_v - S_V;$$

$$\text{or } \frac{(6{\cdot}22)^3}{64} S = S_{1049} - S_{1383};$$

$$\text{or } {\cdot}6045S = 3595{\cdot}6 - 1432{\cdot}4;$$

$$\text{or } S = \frac{2163{\cdot}2}{{\cdot}6045} = 3578 \text{ ft.}$$

i.e., the distance of the crest of the parapet from the gun is 3578 ft.

Table showing the Resistance of the Air in lbs. to Elongated Projectiles with Ogival Heads, from 1 in. to 15 ins. in diameter, at velocities of from 900 f.s. to 1700 f.s.

Velocity.	Diameter of projectile.															Velocity.
	1 in.	2 in.	3 in.	4 in.	5 in.	6 in.	7 in.	8 in.	9 in.	10 in.	11 in.	12 in.	13 in.	14 in.	15 in.	
f.s.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	f.s.
900	1·4584	5·8	13·1	23·3	36·5	52·5	71·5	93·3	118	146	176	210	246	286	328	900
950	1·7951	7·2	16·2	28·7	44·9	64·6	88	115	145	180	217	258	303	352	404	950
1000	2·3299	9·3	21	37·3	58·2	83·9	114	149	189	233	282	336	394	457	524	1000
1050	3·3373	13·3	30	53·4	83·4	120	164	214	270	334	404	481	564	654	751	1050
1100	4·3829	17·5	39	70	110	158	215	281	355	438	530	631	741	859	986	1100
1150	5·1121	20·4	46	82	128	184	250	327	414	511	619	736	864	1002	1150	1150
1200	5·8457	23·4	53	94	146	210	286	374	474	585	707	842	988	1146	1315	1200
1250	6·5952	26·4	59	106	165	237	323	422	534	660	798	950	1115	1293	1484	1250
1300	7·3641	29·5	66	118	184	265	361	471	596	736	891	1060	1245	1443	1657	1300
1350	8·1322	32·5	73	130	203	293	398	520	659	813	984	1171	1374	1594	1830	1350
1400	8·8650	35·5	80	142	222	319	434	567	718	887	1073	1277	1498	1738	1995	1400
1450	9·5556	38·2	86	153	239	344	468	612	774	956	1156	1376	1615	1873	2150	1450
1500	10·1902	40·8	92	163	255	367	499	652	825	1019	1233	1467	1722	1997	2293	1500
1550	10·7585	43	97	172	269	387	527	689	871	1076	1302	1549	1818	2109	2421	1550
1600	11·3245	45·3	102	181	283	408	555	725	917	1132	1370	1631	1914	2220	2548	1600
1650	11·9174	47·7	107	191	298	429	584	763	965	1192	1442	1716	2014	2336	2681	1650
1700	12·8050	51·2	115	205	320	461	627	820	1037	1281	1549	1844	2164	2510	2881	1700

NOTE.—The values of K for spherical shot have also been determined experimentally for velocities varying from 850 to 2150 f.s. Also tables of S_v and T_v have been calculated for spherical shot. (*Vide* "Proceedings, R.A. Institution," Vols. VII. and VIII.)

TABLE I.

Distance and Velocity Table. $\frac{a^2}{w} S = S_v - S_V$.

*v.	0	1	2	3	4	5	6	7	8	9
f.s.	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.
50	19604·0	19542·0	19480·3	19418·6	19357·2	19296·1	19235·2	19174·6	19114·3	19054·3
51	18994·6	18935·1	18875·8	18816·7	18757·8	18699·1	18640·7	18582·6	18524·6	18466·8
52	18409·2	18351·9	18294·9	18238·0	18181·3	18124·8	18068·5	18012·5	17956·7	17901·1
53	17845·7	17790·6	17735·6	17680·8	17626·2	17571·9	17517·8	17463·9	17410·1	17356·5
54	17303·2	17250·1	17197·2	17144·4	17091·8	17039·4	16987·1	16935·0	16883·1	16831·5
55	16780·2	16729·1	16678·1	16627·2	16576·5	16526·0	16475·7	16425·6	16375·6	16325·8
56	16276·2	16226·9	16177·7	16128·6	16079·6	16030·8	15982·2	15933·8	15885·6	15837·6
57	15789·8	15742·1	15694·6	15647·2	15600·0	15552·9	15506·0	15459·3	15412·7	15366·3
58	15320·1	15274·0	15228·1	15182·3	15136·7	15091·2	15045·9	15000·7	14955·7	14910·9
59	14866·2	14821·7	14777·3	14733·1	14689·0	14645·1	14601·3	14557·7	14514·2	14470·8
60	14427·7	14384·8	14341·9	14299·1	14256·4	14213·8	14171·3	14129·0	14086·9	14045·0
61	14003·3	13961·7	13920·2	13878·9	13837·6	13796·4	13755·3	13714·4	13673·7	13633·2
62	13592·8	13552·4	13512·2	13472·1	13432·2	13392·4	13352·7	13313·1	13273·7	13234·4
63	13195·2	13156·1	13117·3	13078·5	13039·8	13001·2	12962·8	12924·5	12886·3	12848·2
64	12810·2	12772·3	12734·6	12697·0	12695·5	12622·1	12584·8	12547·6	12510·6	12473·7
65	12436·9	12400·2	12363·7	12327·2	12290·8	12254·5	12218·3	12182·3	12146·4	12110·6
66	12074·9	12039·3	12003·9	11968·5	11933·2	11898·0	11863·0	11828·1	11793·2	11758·4
67	11723·7	11689·1	11654·7	11620·4	11586·2	11552·1	11518·1	11484·2	11450·4	11416·7
68	11383·0	11349·4	11316·0	11282·7	11249·5	11216·3	11183·3	11150·4	11117·5	11084·7
69	11052·0	11019·4	10987·0	10954·6	10922·3	10890·1	10857·9	10825·9	10794·0	10762·2
70	10730·5	10698·9	10667·4	10635·9	10604·5	10573·2	10542·0	10510·9	10479·9	10448·9
71	10418·0	10387·2	10356·5	10326·0	10295·5	10265·1	10234·8	10204·6	10174·4	10144·3
72	10114·3	10084·4	10054·6	10024·9	9995·2	9965·6	9936·1	9906·7	9877·4	9848·1
73	9818·9	9789·8	9760·8	9731·9	9703·0	9674·2	9645·5	9616 9	9588·3	9559·8
74	9531·4	9503·1	9474·9	9446·7	9418·6	9390·6	9362·6	9334·7	9306·9	9279·2
75	9251·6	9224·1	9196·6	9169·2	9141·8	9114·5	9087·3	9060·2	9033·1	9006·1
76	8979·2	8952·4	8925·7	8899·0	8872·3	8845·7	8819·2	8792·8	8766·4	8740·1
77	8713·9	8687·7	8661·6	8635·6	8609·7	8583·8	8558·0	8532·3	8506·0	8480·9
78	8455·3	8429·8	8404·4	8379·0	8353·7	8328·5	8303·3	8278·2	8253·2	8228·2
79	8203·3	8178·5	8153·7	8129·0	8104·3	8079·7	8055·2	8030·8	8006·4	7982·0
80	7957·7	7933·5	7909·3	7885·2	7861·1	7837·1	7813·2	7789·3	7765·5	7741·7
81	7718·0	7694·4	7670·8	7647·3	7623·8	7600·4	7577·0	7553·7	7530·5	7507·3
82	7484·2	7461·1	7438·1	7415·1	7392·2	7369·4	7346·6	7323·9	7301·2	7278·6
83	7256·0	7233·5	7211·1	7188·7	7166·3	7144·0	7121·8	7099·6	7077·5	7055·4
84	7033·4	7011·4	6989·5	6967·6	6945·8	6924·0	6902·2	6880·5	6858·9	6837·3
85	6815·8	6794·4	6773·0	6751·6	6730·3	6709·0	6687·8	6666·6	6645·5	6624·5
86	6603·5	6582·5	6561·6	6540·7	6519·9	6499·1	6478·4	6457·7	6437·1	6416·5
87	6395·9	6375·4	6355·0	6334·6	6314·2	6293·9	6273·7	6253·5	6233·3	6213·2
88	6193·1	6173·1	6153·1	6133·2	6113·3	6093·4	6073·6	6053·8	6034·1	6014·4
89	5994·8	5975·2	5955·7	5936·2	5916·7	5897·3	5877·9	5858·6	5839·3	5820·0
90	5800·8	5781·6	5762·5	5743·5	5724·5	5705·6	5686·7	5667·9	5649·2	5630·5
91	5611·8	5593·2	5574·7	5556·2	5537·7	5519·3	5500·9	5482·6	5464·4	5446·2
92	5428·1	5410·0	5392·0	5374·1	5356 2	5338·3	5320·5	5302·8	5285·1	5267·5
93	5249·9	5232·4	5215·0	5197·6	5180·2	5162·9	5145·7	5128·5	5111·4	5094·3
94	5077·3	5060·4	5043·5	5026·6	5009·8	4993·0	4976·3	4959·6	4943·0	4926·5
95	4910·0	4893·6	4877·3	4861·0	4844·7	4828·5	4812·4	4796·3	4780·3	4764·4
96	4748·5	4732·7	4716·9	4701·2	4685·5	4669·9	4654·4	4638·9	4623·5	4608·1
97	4592·8	4577·6	4562·4	4547·3	4532·2	4517·2	4502·3	4487·4	4472·6	4457·8
98	4443·1	4428·5	4413·9	4399·4	4384·9	4370·5	4356·2	4342·0	4327·8	4313·6
99	4299·5	4285·4	4271·4	4257·5	4243·7	4230·0	4216·4	4202·8	4189·3	4175·8
100	4162·4	4149·1	4135·9	4122·8	4109·7	4096·7	4083·8	4070·9	4058·1	4045·3
101	4032·6	4020·0	4007·5	3995·0	3982·5	3970·1	3957·8	3945·6	3933·5	3921·4
102	3909·5	3897·7	3886·0	3874·3	3862·7	3851·2	3839·8	3828·5	3817·3	3806·1
103	3795·0	3784·0	3773·0	3762·1	3751·2	3740·4	3729·6	3718·9	3708·3	3697·7
104	3687·2	3676·8	3666·4	3656·1	3645·8	3635·6	3625·5	3615·5	3605·5	3595·6
105	3585·8	3576·1	3566·5	3556·9	3547·3	3537·8	3528·4	3519·1	3509·8	3500·6
106	3491·4	3482·3	3473·2	3464·2	3455·2	3446·3	3437·5	3428·7	3419·9	3411·1

* In the left hand vertical column the figures are hundreds and tens; thus, 51 means five hundred and ten feet seconds. The unit figure of velocity is given in the top horizontal column; thus, if velocity (v) = 504 f.s., then $S_v = S_{504} = 19357{\cdot}2$ ft., or, conversely, if $S_v = 19357{\cdot}2$ ft., then $v = 504$ f.s.

TABLE I.—*(Continued).*

v.	0	1	2	3	4	5	6	7	8	9
f.s.	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.
107	3402·4	3393·8	3385·2	3376·7	3368·1	3359·6	3351·2	3342·8	3334·4	3326·0
108	3317·7	3309·5	3301·3	3293·1	3284·9	3276·7	3268·6	3260·5	3252·4	3244·3
109	3236·3	3228·4	3220·4	3212·5	3204·5	3196·6	3188·7	3180·9	3173·1	3165·3
110	3157·5	3149·7	3141·9	3134·2	3126·5	3118·7	3111·0	3103·3	3095·6	3087·9
111	3080·3	3072·8	3065·2	3057·6	3050·1	3042·5	3035·0	3027·5	3020·0	3012·6
112	3005·1	2997·7	2990·3	2982·9	2975·5	2968·1	2960·8	2953·4	2946·1	2938·7
113	2931·4	2924·1	2916·8	2909·6	2902·4	2895·2	2888·1	2880·9	2873·7	2866·6
114	2859·4	2852·3	2845·2	2838·1	2831·0	2823·9	2816·9	2809·8	2802·8	2795·8
115	2788·8	2781·8	2774·9	2767·9	2761·0	2754·0	2747·1	2740·2	2733·3	2726·5
116	2719·6	2712·8	2705·9	2699·1	2692·3	2685·5	2678·7	2672·0	2665·2	2658·5
117	2651·7	2645·0	2638·3	2631·6	2625·0	2618·3	2611·6	2605·0	2598·4	2591·8
118	2585·2	2578·6	2572·1	2565·5	2558·9	2552·4	2545·8	2539·3	2532·8	2526·3
119	2519·8	2513·4	2506·9	2500·4	2494·0	2487·5	2481·1	2474·7	2468·3	2461·9
120	2455·5	2449·1	2442·8	2436·4	2430·1	2423·8	2417·4	2411·1	2404·8	2398·5
121	2392·2	2386·0	2379·7	2373·5	2367·3	2361·0	2354·8	2348·6	2342·4	2336·2
122	2330·0	2323·9	2317·7	2311·6	2305·5	2299·3	2293·2	2287·1	2281·0	2274·9
123	2268·8	2262·7	2256·7	2250·7	2244·6	2238·6	2232·6	2226·6	2220·6	2214·6
124	2208·6	2202·6	2196·7	2190·7	2184·8	2178·8	2172·9	2166·9	2161·0	2155·1
125	2149·2	2143·3	2137·5	2131·6	2125·8	2119·9	2114·1	2108·2	2102·4	2096·6
126	2090·8	2085·0	2079·2	2073·4	2067·6	2061·9	2056·1	2050·4	2044·7	2038·9
127	2033·2	2027·5	2021·8	2016·1	2010·5	2004·8	1999·1	1993·5	1987·8	1982·2
128	1976·5	1970·8	1965·2	1959·6	1954·0	1948·4	1942·8	1937·2	1931·7	1926·1
129	1920·5	1915·0	1909·4	1903·9	1898·4	1892·8	1887·3	1881·8	1876·3	1870·8
130	1865·3	1859·8	1854·4	1848·9	1843·5	1838·0	1832·6	1827·2	1821·7	1816·3
131	1810·9	1805·5	1800·1	1794·7	1789·3	1783·9	1778·5	1773·1	1767·8	1762·4
132	1757·0	1751·7	1746·3	1741·0	1735·7	1730·4	1725·1	1719·8	1714·5	1709·2
133	1703·9	1698·6	1693·4	1688·1	1682·9	1677·6	1672·4	1667·1	1661·9	1656·7
134	1651·4	1646·2	1641·0	1635·8	1630·6	1625·4	1620·2	1615·0	1609·9	1604·7
135	1599·5	1594·4	1589·2	1584·1	1578·9	1573·8	1568·7	1563·5	1558·4	1553·3
136	1548·2	1543·1	1538·0	1533·0	1527·9	1522·8	1517·8	1512·7	1507·7	1502·6
137	1497·6	1492·5	1487·5	1482·5	1477·4	1472·4	1467·4	1462·3	1457·3	1452·3
138	1447·3	1442·3	1437·3	1432·4	1427·4	1422·4	1417·5	1412·5	1407·6	1402·6
139	1397·7	1392·7	1387·8	1382·9	1377·9	1373·0	1368·1	1363·1	1358·2	1353·3
140	1348·4	1343·5	1338·6	1333·7	1328·8	1323·9	1319·0	1314·1	1309·3	1304·4
141	1299·5	1294·7	1289·8	1284·9	1280·1	1275·2	1270·4	1265·6	1260·8	1255·9
142	1251·1	1246·3	1241·5	1236·6	1231·8	1227·0	1222·2	1217·4	1212·6	1207·8
143	1203·0	1198·2	1193·4	1188·7	1183·9	1179·1	1174·4	1169·6	1164·9	1160·2
144	1155·4	1150·7	1145·9	1141·2	1136·4	1131·7	1126·9	1122·2	1117·5	1112·7
145	1108·0	1103·3	1098·6	1093·9	1089·2	1084·5	1079·8	1075·1	1070·4	1065·8
146	1061·1	1056·4	1051·7	1047·1	1042·4	1037·7	1033·1	1028·4	1023·7	1019·1
147	1014·4	1009·8	1005·1	1000·5	995·8	991·2	986·6	981·9	977·3	972·7
148	968·0	963·4	958·8	954·1	949·5	944·9	940·3	935·7	931·1	926·5
149	921·9	917·3	912·7	908·1	903·5	898·9	894·3	889·7	885·1	880·6
150	876·0	871·4	866·9	862·3	857·8	853·2	848·6	844·1	839·5	835·0
151	830·4	825·9	821·3	816·8	812·2	807·7	803·2	798·6	794·1	789·5
152	785·0	780·5	775·9	771·4	766·9	762·3	757·8	753·3	748·7	744·2
153	739·7	735·2	730·7	726·2	721·7	717·2	712·7	708·2	703·7	699·2
154	694·7	690·2	685·7	681·2	676·8	672·3	667·8	663·3	658·8	654·4
155	649·9	645·4	640·9	636·5	632·0	627·5	623·1	618·6	614·2	609·7
156	605·3	600·8	596·4	591·9	587·5	583·0	578·6	574·1	569·7	565·2
157	560·8	556·4	551·9	547·5	543·0	538·6	534·2	529·7	525·3	520·9
158	516·4	512·0	507·6	503·1	498·7	494·3	489·9	485·5	481·1	476·7
159	472·3	467·9	463·5	459·1	454·7	450·3	445·9	441·5	437·1	432·7
160	428·3	423·9	419·5	415·1	410·7	406·4	402·0	397·6	393·2	388·9
161	384·5	380·1	375·8	371·4	367·1	362·7	358·3	354·0	349·6	345·3
162	340·9	336·5	332·2	327·8	323·5	319·1	314·8	310·4	306·1	301·7
163	297·4	293·1	288·7	284·4	280·0	275·7	271·4	267·0	262·7	258·4
164	254·0	249·7	245·4	241·0	236·7	232·4	228·1	223·8	219·5	215·2
165	210·9	206·6	202·3	198·0	193·7	189·4	185·1	180·8	176·5	172·3
166	168·0	163·7	159·5	155·2	150·9	146·7	142·4	138·2	134·0	129·7
167	125·5	121·3	117·0	112·8	108·6	104·3	100·1	95·9	91·7	87·5
168	83·3	79·1	74·9	70·7	66·5	62·3	58·1	53·9	49·8	45·6
169	41·4	37·2	33·1	28·9	24·8	20·7	16·5	12·3	8·2	4·1
170	0·0									

Table II.

Time and Velocity Table. $\frac{d^2}{w} T = T_v - T_V$.

*v.	0	1	2	3	4	5	6	7	8	9
f.s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.
50	26·5083	26·3844	26·2612	26·1388	26·0172	25·8963	25·7762	25·6567	25·5379	25·4198
51	25·3024	25·1857	25·0697	24·9543	24·8396	24·7255	24·6121	24·4994	24·3873	24·2759
52	24·1652	24·0552	23·9457	23·8368	23·7285	23·6209	23·5139	23·4075	23·3017	23·1965
53	23·0919	22·9879	22·8845	22·7817	22·6795	22·5778	22·4766	22·3760	22·2760	22·1766
54	22·0778	21·9795	21·8817	21·7844	21·6877	21·5915	21·4958	21·4006	21·3060	21·2119
55	21·1184	21·0254	20·9328	20·8407	20·7491	20·6580	20·5674	20·4773	20·3877	20·2986
56	20·2100	20·1218	20·0341	19·9468	19·8600	19·7737	19·6879	19·6025	19·5175	19·4330
57	19·3489	19·2653	19·1821	19·0994	19·0171	18·9352	18·8537	18·7726	18·6920	18·6118
58	18·5320	18·4527	18·3738	18·2952	18·2170	18·1392	18·0619	17·9849	17·9083	17·8321
59	17·7563	17·6809	17·6059	17·5312	17·4569	17·3830	17·3094	17·2362	17·1634	17·0910
60	17·0190	16·9474	16·8761	16·8051	16·7344	17·6640	16·5940	16·5244	16·4551	16·3862
61	16·3176	16·2494	16·1815	16·1139	16·0466	15·9797	15·9131	15·8468	15·7809	15·7153
62	15·6500	15·5851	15·5204	15·4560	15·3919	15·3281	15·2646	15·2014	15·1386	15·0761
63	15·0139	14·9520	14·8903	14·8289	14·7678	14·7070	14·6465	14·5863	14·5264	14·4668
64	14·4074	14·3483	14·2895	14·2309	14·1726	14·1146	14·0569	13·9994	13·9422	13·8853
65	13·8286	13·7722	13·7160	13·6601	13·6045	13·5491	13·4940	13·4391	13·3845	13·3301
66	13·2759	13·2220	13·1684	13·1150	13·0618	13·0089	12·9562	12·9038	12·8516	12·7996
67	12·7479	12·6964	12·6451	12·5940	12·5432	12·4926	12·4422	12·3920	12·3421	12·2924
68	12·2429	12·1937	12·1447	12·0958	12·0471	11·9986	11·9504	11·9025	11·8547	11·8071
69	11·7597	11·7125	11·6655	11·6188	11·5723	11·5260	11·4799	11·4340	11·3882	11·3426
70	11·2972	11·2520	11·2070	11·1622	11·1176	11·0732	11·0290	10·9850	10·9412	10·8975
71	10·8540	10·8107	10·7675	10·7246	10·6819	10·6394	10·5970	10·5548	10·5127	10·4708
72	10·4291	10·3875	10·3461	10·3049	10·2639	10·2232	10·1826	10·1421	10·1018	10·0616
73	10·0216	9·9816	9·9420	9·9025	9·8632	9·8241	9·7851	9·7462	9·7075	9·6689
74	9·6305	9·5922	9·5541	9·5162	9·4784	9·4408	9·4034	9·3661	9·3289	9·2919
75	9·2550	9·2183	9·1817	9·1453	9·1090	9·0728	9·0368	9·0009	8·9652	8·9296
76	8·8941	8·8588	8·8236	8·7886	8·7537	8·7189	8·6843	8·6498	8·6155	8·5813
77	8·5472	8·5133	8·4795	8·4458	8·4122	8·3788	8·3455	8·3123	8·2793	8·2464
78	8·2136	8·1809	8·1484	8·1160	8·0837	8·0515	8·0195	7·9876	7·9558	7·9241
79	7·8926	7·8612	7·8299	7·7987	7·7676	7·7366	7·7058	7·6751	7·6445	7·6139
80	7·5835	7·5532	7·5231	7·4931	7·4631	7·4333	7·4036	7·3741	7·3446	7·3152
81	7·2859	7·2567	7·2276	7·1986	7·1698	7·1411	7·1125	7·0840	7·0556	7·0272
82	6·9990	6·9709	6·9429	6·9150	6·8871	6·8594	6·8318	6·8043	6·7769	6·7496
83	6·7224	6·6953	6·6683	6·6414	6·6146	6·5878	6·5612	6·5347	6·5083	6·4819
84	6·4557	6·4296	6·4036	6·3776	6·3517	6·3259	6·3002	6·2746	6·2491	6·2236
85	6·1983	6·1731	6·1480	6·1229	6·0979	6·0730	6·0482	6·0236	5·9990	5·9744
86	5·9499	5·9255	5·9012	5·8770	5·8529	5·8289	5·8049	5.7810	5·7572	5·7335
87	5·7099	5·6864	5·6629	5·6395	5·6162	5·5930	5·5699	5·5468	5·5238	5·5009
88	5·4781	5·4553	5·4326	5·4100	5·3875	5·3651	5·3428	5·3205	5·2983	5·2762
89	5·2541	5·2321	5·2102	5·1884	5·1666	5·1449	5·1233	5·1018	5·0803	5·0589
90	5·0375	5·0163	4·9952	4·9741	4·9531	4·9321	4·9112	4·8904	4·8697	4·8491
91	4·8286	4·8082	4·7878	4·7675	4·7473	4·7272	4·7072	4·6873	4·6674	4·6476
92	4·6279	4·6083	4·5888	4·5693	4·5499	4·5306	4·5114	4·4923	4·4732	4·4542
93	4·4353	4·4165	4·3978	4·3791	4·3605	4·3421	4·3236	4·3053	4·2870	4·2688
94	4·2506	4·2326	4·2147	4·1968	4·1790	4·1612	4·1436	4·1260	4·1085	4·0911
95	4·0738	4·0565	4·0393	4·0222	4·0052	3·9883	3·9714	3·9546	3·9379	3·9213
96	3·9047	3·8882	3·8718	3·8555	3·8393	3·8231	3·8070	3·7910	3·7751	3·7592
97	3·7434	3·7277	3·7121	3·6966	3·6811	3·6657	3·6504	3·6352	3·6200	3·6049
98	3·5899	3·5750	3·5602	3·5455	3·5308	3·5162	3·5017	3·4873	3·4729	3·4586
99	3·4444	3·4303	3·4163	3·4024	3·3885	3·3747	3·3610	3·3474	3·3339	3·3204
100	3·3070	3·2937	3·2805	3·2674	3·2544	3·2414	3·2285	3·2157	3·2030	3·1904
101	3·1778	3·1653	3·1529	3·1406	3·1284	3·1163	3·1042	3·0922	3·0803	3·0685
102	3·0567	3·0451	3·0335	3·0220	3·0106	2·9992	2·9879	2·9767	2·9656	2·9546
103	2·9436	2·9328	2·9220	2·9113	2·9007	2·8902	2·8798	2·8694	2·8591	2·8489
104	2·8388	2·8288	2·8188	2·8089	2·7991	2·7894	2·7798	2·7703	2·7608	2·7514
105	2·7420	2·7328	2·7236	2·7144	2·7053	2·6963	2·6874	2·6785	2·6697	2·6610
106	2·6523	2·6437	2·6351	2·6266	2·6182	2·6098	2·6015	2·5932	2·5850	2·5768

Vide note, Table I. EXAMPLE.—If $v = 504$ f.s., $T_v = T_{504} = 26{\cdot}0172$ secs.; or if $T_v = 26{\cdot}0172$ secs., $v = 504$ f.s.

TABLE II.—(*Continued.*)

v.	0	1	2	3	4	5	6	7	8	9
f.s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.
107	2·5687	2·5606	2·5526	2·5446	2·5366	2·5287	2·5208	2·5130	2·5052	2·4975
108	2·4898	2·4821	2·4745	2·4669	2·4593	2·4518	2·4443	2·4369	2·4295	2·4221
109	2·4147	2·4074	2·4001	2·3928	2·3855	2·3783	2·3711	2·3640	2·3568	2·3497
110	2·3426	2·3355	2·3285	2·3214	2·3144	2·3074	2·3005	2·2936	2·2867	2·2798
111	2·2729	2·2661	2·2592	2·2524	2·2456	2·2388	2·2321	2·2254	2·2187	2·2120
112	2·2053	2·1987	2·1921	2·1855	2·1789	2·1723	2·1658	2·1593	2·1528	2·1464
113	2·1399	2·1335	2·1271	2·1207	2·1134	2·1079	2·1016	2·0953	2·0890	2·0827
114	2·0764	2·0702	2·0639	2·0577	2·0515	2·0453	2·0392	2·0330	2·0269	2·0208
115	2·0147	2·0087	2·0026	1·9966	1·9905	1·9845	1·9785	1·9726	1·9667	1·9607
116	1·9548	1·9489	1·9431	1·9372	1·9314	1·9255	1·9197	1·9139	1·9081	1·9024
117	1·8966	1·8909	1·8852	1·8795	1·8738	1·8681	1·8625	1·8568	1·8512	1·8455
118	1·8399	1·8343	1·8287	1·8232	1·8176	1·8121	1·8066	1·8011	1·7956	1·7901
119	1·7847	1·7792	1·7738	1·7684	1·7630	1·7576	1·7523	1·7469	1·7416	1·7362
120	1·7309	1·7256	1·7203	1·7150	1·7098	1·7045	1·6992	1·6940	1·6888	1·6836
121	1·6784	1·6732	1·6681	1·6629	1·6578	1·6526	1·6475	1·6424	1·6373	1·6323
122	1·6272	1·6222	1·6172	1·6121	1·6071	1·6021	1·5971	1·5922	1·5872	1·5823
123	1·5773	1·5724	1·5675	1·5626	1·5577	1·5528	1·5479	1·5431	1·5382	1·5334
124	1·5285	1·5237	1·5189	1·5141	1·5093	1·5045	1·4997	1·4950	1·4903	1·4855
125	1·4808	1·4761	1·4714	1·4667	1·4620	1·4574	1·4527	1·4481	1·4435	1·4388
126	1·4342	1·4296	1·4250	1·4204	1·4158	1·4113	1·4068	1·4022	1·3977	1·3932
127	1·3887	1·3842	1·3797	1·3752	1·3708	1·3663	1·3619	1·3574	1·3530	1·3486
128	1·3442	1·3398	1·3354	1·3310	1·3267	1·3223	1·3180	1·3137	1·3093	1·3050
129	1·3007	1·2964	1·2921	1·2878	1·2835	1·2792	1·2749	1·2707	1·2665	1·2622
130	1·2580	1·2538	1·2496	1·2454	1·2412	1·2370	1·2329	1·2287	1·2246	1·2204
131	1·2163	1·2122	1·2080	1·2039	1·1998	1·1957	1·1916	1·1876	1·1835	1·1794
132	1·1754	1·1713	1·1673	1·1633	1·1592	1·1552	1·1512	1·1472	1·1432	1·1393
133	1·1353	1·1313	1·1274	1·1234	1·1195	1·1156	1·1116	1·1077	1·1038	1·0999
134	1·0960	1·0922	1·0883	1·0844	1·0806	1·0767	1·0729	1·0690	1·0652	1·0614
135	1·0575	1·0537	1·0499	1·0461	1·0423	1·0385	1·0347	1·0309	1·0272	1·0234
136	1·0196	1·0159	1·0121	1·0084	1·0047	1·0009	·9972	·9935	·9898	·9861
137	·9824	·9787	·9751	·9714	·9678	·9641	·9605	·9568	·9532	·9496
138	·9459	·9423	·9387	·9351	·9315	·9279	·9243	·9207	·9172	·9136
139	·9100	·9065	·9029	·8994	·8959	·8923	·8888	·8853	·8818	·8782
140	·8747	·8712	·8677	·8642	·8607	·8572	·8538	·8503	·8468	·8434
141	·8399	·8364	·8330	·8296	·8261	·8227	·8193	·8159	·8125	·8091
142	·8057	·8023	·7989	·7956	·7922	·7888	·7854	·7821	·7787	·7754
143	·7720	·7687	·7654	·7620	·7587	·7554	·7521	·7488	·7454	·7421
144	·7388	·7355	·7322	·7289	·7256	·7223	·7191	·7158	·7125	·7093
145	·7060	·7028	·6996	·6963	·6931	·6899	·6867	·6834	·6802	·6770
146	·6738	·6706	·6674	·6642	·6610	·6578	·6546	·6514	·6483	·6451
147	·6419	·6387	·6356	·6324	·6293	·6261	·6230	·6198	·6167	·6136
148	·6104	·6073	·6042	·6011	·5980	·5949	·5918	·5887	·5856	·5825
149	·5794	·5763	·5732	·5701	·5671	·5640	·5609	·5579	·5548	·5518
150	·5487	·5457	·5426	·5396	·5366	·5335	·5305	·5275	·5244	·5214
151	·5184	·5154	·5124	·5094	·5064	·5034	·5004	·4974	·4944	·4914
152	·4884	·4854	·4824	·4795	·4765	·4735	·4705	·4676	·4646	·4617
153	·4587	·4558	·4528	·4499	·4470	·4440	·4411	·4382	·4353	·4323
154	·4294	·4265	·4236	·4207	·4178	·4149	·4120	·4091	·4062	·4033
155	·4004	·3975	·3946	·3918	·3889	·3860	·3831	·3803	·3774	·3745
156	·3717	·3688	·3660	·3632	·3603	·3575	·3547	·3518	·3490	·3462
157	·3433	·3405	·3377	·3349	·3320	·3292	·3264	·3236	·3207	·3179
158	·3151	·3123	·3095	·3067	·3039	·3012	·2984	·2956	·2928	·2901
159	·2873	·2845	·2818	·2790	·2763	·2735	·2707	·2680	·2652	·2625
160	·2597	·2570	·2542	·2515	·2488	·2460	·2433	·2406	·2378	·2351
161	·2324	·2297	·2270	·2243	·2216	·2189	·2162	·2135	·2108	·2081
162	·2054	·2027	·2000	·1973	·1947	·1920	·1893	·1866	·1839	·1813
163	·1786	·1759	·1733	·1706	·1680	·1653	·1627	·1600	·1574	·1548
164	·1521	·1495	·1469	·1442	·1416	·1390	·1364	·1338	·1311	·1285
165	·1259	·1233	·1207	·1181	·1155	·1129	·1103	·1077	·1051	·1026
166	·1000	·0974	·0949	·0923	·0897	·0872	·0846	·0821	·0795	·0770
167	·0745	·0720	·0695	·0669	·0644	·0619	·0594	·0569	·0543	·0518
168	·0493	·0468	·0443	·0418	·0393	·0368	·0343	·0318	·0294	·0269
169	·0244	·0220	·0195	·0171	·0146	·0122	·0098	·0073	·0049	·0024
170	·0000									

CHAPTER VI.

TRAJECTORIES.

Unresisted Projectile.—Symbols used.—Equations of Motion of an Unresisted Projectile.—Equation of the Trajectory.—Elementary Discussion of the Conditions of Motion.—The Range on Horizontal Plane in terms of Muzzle Velocity and Angle of Departure.—The Time of Flight on a Horizontal Plane in terms of Muzzle Velocity and Angle of Departure.—The Projectile reaches its Highest Point at one-half of its Range on the Horizontal Plane, and in one-half of its total Time of Flight.—The Height of the Trajectory at any given time.—The Maximum Height of Trajectory.—The Inclination of the Projectile to the Horizontal Line in the Plane of the Trajectory.—Approximate Solution of Low-Angle Trajectories.—Angle of Descent.—Maximum Height of Trajectory.—Method of Finding the "Dangerous Distance," or the Distance in which an Object of given height would be struck, under given conditions.—Method of Finding Height of Trajectory at given Distances.—Motion of a Projectile in the Air.—Equations of Motion.—The Velocity at the Vertex of the Trajectory.—Bashforth's Method of Solution.—Niven's Method of Solution.—Practical Examples.

Unresisted projectile.

BEFORE discussing the motion of a projectile in the air, it will be necessary first to consider the simpler case in which the resistance of the air is neglected, and the projectile is supposed to move without any resistance.

Symbols used.

The following symbols will be used in this branch of the subject:—

v for the velocity of the projectile at any point of the trajectory,
V " muzzle velocity,
ϕ " inclination of the direction of motion to the horizontal line in the plane of the trajectory,
α " angle of departure,
t " time,
x " horizontal distance from some fixed point,
y " vertical distance,
g " acceleration due to gravity.

Equations of motion of an unresisted projectile.

The equations of motion, then, in a horizontal and vertical direction will be

$$\frac{d^2x}{dt^2} = 0, \text{ and } \frac{d^2y}{dt^2} = -g.$$

Integrating,

$$\frac{dx}{dt} = \text{const.} = V\cos\alpha,$$

$$\text{and } \frac{dy}{dt} = \text{const.} - gt = V\sin\alpha - gt.$$

Integrating again,

$$x = Vt \cos \alpha, \quad \ldots\ldots(1)$$

$$y = Vt \sin \alpha - \tfrac{1}{2}gt^2; \quad \ldots\ldots(2)$$

Equation of the trajectory.

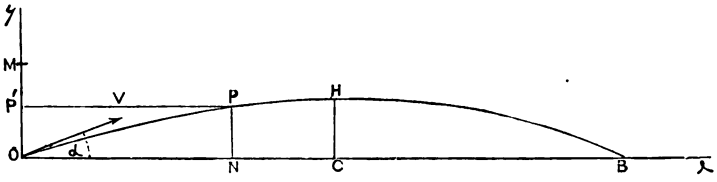
or, eliminating t between equations (1) and (2),

$$y = x \tan \alpha - \frac{gx^2}{2V^2 \cos^2 \alpha}; \quad \ldots\ldots(3)$$

which is the equation of the trajectory of an unresisted projectile.

Elementary discussion of the conditions of motion.

The same results may be obtained more simply by considering the conditions of motion. Thus, suppose (x, y) the co-ordinates of the



projectile (P) at time t. Then

$$ON = x,$$

$$NP = y,$$

horizontal velocity at muzzle $= V \cos \alpha$;

and since there is no resistance, and gravity acts only in a *vertical* direction,

the horizontal velocity at any point in the trajectory is the same, and equal to $V \cos \alpha$.

Again,

vertical velocity at muzzle $= V \sin \alpha$.

This vertical velocity is diminished by gravity acting vertically downwards—*i.e.*, by gt in time t—so that

vertical velocity at any time, t, is equal to $V \sin \alpha - gt$.

The horizontal velocity being constant, and equal to $V \cos \alpha$, it follows that

the horizontal distance (ON) traversed by the projectile in time t

$= V \cos \alpha \times t$;

or $x = Vt \cos \alpha$. $\ldots\ldots(1)$

In the same way, if gravity were not acting,

the vertical distance traversed in time t would be

$$V \sin \alpha \times t;$$

or $$OM = Vt \sin \alpha.$$

This distance is diminished by the distance that would be traversed by the projectile falling freely under gravity in time t.

This latter distance, $MP' = \frac{1}{2}gt^2$. (*Vide* Elementary Treatises on Dynamics). Hence,

$$PN = P'O = OM - MP';$$

$$\text{or } y = Vt \sin \alpha - \tfrac{1}{2}gt^2. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

By eliminating t, as before, the equation of the trajectory of an unresisted projectile is

$$y = x \tan \alpha - \frac{gx^2}{2V^2 \cos^2 \alpha}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The trajectory is a parabola, with the axis vertical, as in the figure.

To find the range on a horizontal plane passing through the muzzle of the gun, let $y = 0$ in equation (3), then

The range on horizontal plane in terms of muzzle velocity and angle of departure.

$$gx = 2V^2 \sin \alpha \cos \alpha;$$

or if X denote the range,

$$X = \frac{2V^2 \sin \alpha \cos \alpha}{g} = \frac{V^2 \sin 2\alpha}{g}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

It is evident that for a given muzzle velocity, V, the range will be the greatest when $\sin 2\alpha = 1$, *i.e.* when $2\alpha = 90°$ or $\alpha = 45°$.

Again, if T be the time of flight for range X, since horizontal velocity, $V \cos \alpha$, is constant,

The time of flight on a horizontal plane in terms of muzzle velocity and angle of departure.

$$X = VT \cos \alpha;$$

$$\text{or } T = \frac{X}{V \cos \alpha} = \frac{2V^2 \sin \alpha \cos \alpha}{Vg \cos \alpha} = \frac{2V \sin \alpha}{g}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Since the horizontal velocity, $V \cos \alpha$, is constant, it is evident that the projectile will traverse the horizontal distance $\frac{X}{2}$ in time $\frac{T}{2}$.

The projectile reaches its highest point at one-half of its range on the horizontal plane, and in one-half of its total time of flight.

Bisect OB in C, then

$$OC = CB = \frac{X}{2}.$$

Draw CH perpendicular to OB, cutting the trajectory in H.

Since vertical velocity at any time t

$$= V \sin \alpha - gt,$$

therefore vertical velocity at time $\frac{T}{2}$

$$= V \sin \alpha - \frac{gT}{2}.$$

But at highest point this must be zero; so that

$$V \sin \alpha - \frac{gT}{2} = 0,$$

$$\text{or } \frac{T}{2} = \frac{V \sin \alpha}{g};$$

i.e., the projectile has traversed, when it reaches its *highest* point, *one-half of its range* on the horizontal plane in *one-half of the total time of flight.*

The height of the trajectory at any given time.

The height of the trajectory at any time may be found when the whole time of flight is known, thus:

Suppose T the whole time of flight; then from equation (5),

$$V = \frac{gT}{2 \sin \alpha}.$$

Substituting this value of V in equation (2),

$$y = Vt \sin \alpha - \tfrac{1}{2}gt^2 = \frac{g}{2} Tt - \tfrac{1}{2}gt^2;$$

$$\text{or } y = \frac{gt}{2}(T - t). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Referring to the figure (p. 60), if

time over $OB = T$,

" $OP = t$,

" $PB = (T - t) = t'$,

then $PN = \frac{gt}{2}(T - t) = \frac{g}{2} tt'$;

or $PN = \frac{g}{2} \times$ (time over OP) $\times$ (time over PB).

The maximum height of trajectory.

But the height of the trajectory will be a maximum when $t = \frac{T}{2}$; so that substituting in equation (6), it follows that

the *maximum* height of trajectory $= \frac{gT^2}{8}$.

If ϕ be the inclination of the direction of motion of the projectile to the horizontal line in the plane of the trajectory, at any time, t, then

The inclination of the projectile to the horizontal line in the plane of the trajectory.

$$\tan\phi = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\text{vertical velocity at that time}}{\text{horizontal velocity at that time}}; \quad \ldots\ldots\ldots\ldots (7)$$

$$\text{or } \tan\phi = \frac{V\sin\alpha - gt}{V\cos\alpha} = \frac{(T-2t)}{T\cot\alpha}.$$

If β be the angle of descent at time T, then $t = T$; then

$$\tan\beta = \frac{T-2T}{T\cot\alpha} = -\tan\alpha.$$

i.e., with an unresisted projectile the angle of descent is equal to the angle of departure.

The above formulæ, (6) and (7), are very useful for the approximate calculation of the path of a projectile in the air for low-angle trajectories, where the *vertical* velocity may be considered as practically unaffected by the resistance of the air.

Approximate solution of low-angle trajectories.

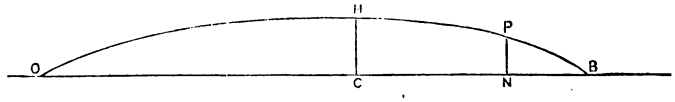
For angles of elevation up to 5° the trajectory for a given range may be determined with considerable accuracy by the use of Tables I. and II. (Chap. V., p. 55).

The remaining velocity at the end of the range is found by Table I. from the equation

$$\frac{d^2}{w}S = S_v - S_V,$$

$$\text{or } S_v = S_V + \frac{d^2}{w}S,$$

on the supposition that the projectile is moving in a horizontal line unaffected by gravity.



Thus, if O be the point of projection, and B the point of impact on the horizontal line through O, then the projectile is supposed to move along OB, and the remaining velocity at B is found from the above equation by means of Table I. Then from equation,

$$\frac{d^2}{w}T = t_v - T_V,$$

and by means of Table II. T (the time of flight over OB) is found.

The action of gravity must now be considered.

For low angles of elevation the vertical velocity of the projectile is small, and the resistance of the air to the projectile in the vertical direction may be neglected; so that all the conditions shown to hold in the vertical plane in the case of an unresisted projectile hold also in this case.

The projectile will ascend approximately for half the time of flight, $= \frac{T}{2}$, and descend for the same time. It will be at its maximum height (H) at the time $\frac{T}{2}$; so that, as before,

$$HC = \frac{gT^2}{8}.$$

In the same way, by Table II. the time over OP and PB may be found, when the velocity at O and the distances ON and NB are known.

Suppose

time over $OP = t$,

" $OB = T$,

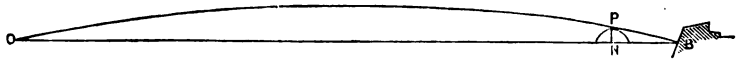
then " $PB = T - t = t'$, suppose;

then height $NP = \frac{g}{2} t\,(T - t)$, from equation (6),

$$= \frac{g}{2} tt'.$$

EXAMPLE 1.—A common shell, fired from a 64-pr. M.L. siege gun with a charge of 12 lbs. of powder, was observed to strike the crest of a traverse in 3·2 secs. Find approximately the range, and what vertical distance below the crest of the traverse the shell would strike an escarp 200 ft. beyond. Muzzle velocity = 1457 f.s., weight of shell = 64 lbs., diameter of shell = 6·22 ins.

The remaining velocity at P can be found thus:



$$T_v = T_V + \frac{d^2}{w} T = T_{1457} + \frac{(6{\cdot}22)^2}{64} \times 3{\cdot}2$$

$$= {\cdot}6834 + 1{\cdot}9344 = 2{\cdot}6178;$$

$$\therefore\ v = 1064 \text{ f.s.} \quad \text{(Table II.)}$$

The distance ON can then be found thus:

$$S = \frac{S_v - S_V}{\frac{d^2}{w}} = \frac{3455{\cdot}2 - 1075{\cdot}1}{{\cdot}6045} = 3937{\cdot}3 \text{ ft.} = ON,$$

the distance of the traverse from the gun.

The velocity at P being known, it is easy to find the velocity at B (200 ft. distant) thus:

$$S_{v'} = S_v + \frac{d^2}{w} S = S_{1064} + \cdot 6045 \times 200$$

$$= 3455 \cdot 2 + 120 \cdot 9 = 3576 \cdot 1;$$

so that

velocity at B or $v' = 1051$ f.s.

Now the time over PB can be found thus:

$$T = \frac{T_{v'} - T_v}{\frac{d^2}{w}} = \frac{T_{1051} - T_{1064}}{\cdot 6045} = \frac{2 \cdot 7328 - 2 \cdot 6182}{\cdot 6045} = \cdot 19 \text{ sec.}$$

Put t' for the time over PB, then

$$t' = \cdot 19 \text{ sec.}$$

But time over $OP = t = 3 \cdot 2$ secs.; so that from equation (6),

$$PN = \frac{g}{2} tt' = \frac{32 \cdot 2}{2} \times 3 \cdot 2 \times \cdot 19 = 9 \cdot 79 \text{ ft.}$$

i.e., the vertical distance below the crest of the traverse that the shell will strike an escarp 200 ft. beyond is 9·79 ft.

The approximate angle of descent at the escarp may be also found. If ϕ is the angle of descent at the escarp, Angle of descent.

$$\tan \phi = \frac{\text{vertical velocity}}{\text{horizontal velocity}}. \quad [\textit{Vide} \text{ equation (7), p. 63}].$$

But if T be the whole time of flight, and the vertical motion be treated as for an unresisted projectile,

$$\text{vertical velocity} = \frac{gT}{2},$$

and horizontal velocity on impact is found from equation

$$T_v = T_V + \frac{d^2}{w} T,$$

so that

$$\tan \phi = \frac{g}{2} \cdot \frac{\text{total time of flight}}{\text{horizontal velocity on impact}};$$

or approximately for small angles,

$$\phi = \frac{180}{\pi} \cdot \frac{g}{2} \cdot \frac{\text{total time of flight}}{\text{horizontal velocity on impact}}.$$

In the above case,

$$T = t + t' = 3 \cdot 2 + \cdot 19 = 3 \cdot 39 \text{ secs.},$$

so that

$$\tan \phi = \frac{32 \cdot 2}{2} \times \frac{3 \cdot 39}{1051} = \cdot 05192;$$

$$\text{or } \phi = 2^\circ\ 58'.$$

Maximum height of trajectory.

The maximum height of the trajectory may also be approximated to thus:—If T be the total time of flight,

$$\text{maximum height of trajectory} = \frac{gT^2}{8}. \quad (\textit{Vide} \text{ p. } 62).$$

$$= \frac{32{\cdot}2}{8}(3{\cdot}39)^2 = 46{\cdot}26 \text{ ft.}$$

Method of finding the "dangerous distance," or the distance in which an object of given height would be struck, under given conditions.

If $y = PN$ is the height of the object to be struck, t the time of flight over OP, and T the time of flight over the given range, OB, then from equation (6),

$$y = \frac{gt}{2}(T - t).$$

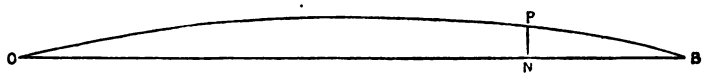
Solving the quadratic, it is found that

$$t = \frac{T}{2} \pm \sqrt{\frac{T^2}{4} - \frac{2y}{g}}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

the upper sign to be taken for the second half of the trajectory. Hence t is known in terms of the total time of flight, and the height of the object fired at.

The time over OP being known, the velocity at P can be found, whence the distance ON is known, and consequently NB—the "dangerous distance"—is known.

Example 2.—The 12·5-in. M.L. gun is fired with a battering charge so as to strike a ship on the water-line at a distance of 3000 ft. What would be the "dangerous distance," or the distance short of 3000 ft. in which the ship whose side was 8·05 ft. above the water-line would be struck somewhere on its side, under the given conditions? Muzzle velocity = 1420 f.s., weight of projectile = 802 lbs., diameter of projectile = 12·42 ins., OB = 3000 ft., PN = 8·05 ft.



The remaining velocity at B can be found thus:

$$S_v = S_V + \frac{d^2}{w}S = S_{1420} + \frac{(12{\cdot}42)^2}{802} \times 3000$$

$$= 1251{\cdot}1 + 576{\cdot}9 = 1828;$$

or $v = 1307$ f.s. (Table I.)

The time over OB can be found thus:

$$T = \frac{T_v - T_V}{\frac{d^2}{w}} = \frac{1{\cdot}2287 - {\cdot}8057}{{\cdot}1923} = 2{\cdot}2 \text{ secs.}$$

Now, to find the time the projectile would be at P, substitute $T = 2{\cdot}2$ and $y = 8{\cdot}05$ in equation (8), or

$$t = \frac{T}{2} + \sqrt{\frac{T^2}{4} - \frac{2y}{32{\cdot}2}} = 1{\cdot}1 + \sqrt{1{\cdot}21 - {\cdot}5};$$

or $t = 1{\cdot}94$ secs.

The remaining velocity at P at the end of 1·94 secs. can be found thus:

$$T_{v'} = \frac{d^2}{w} T + T_V = \frac{(12{\cdot}42)^2}{802} \times 1{\cdot}94 + T_{1420}$$

$$= {\cdot}3731 + {\cdot}8057 = 1{\cdot}1788 \text{ secs.};$$

or $v' = 1319$ f.s.

The distance travelled by the projectile in 1·94 secs. can be found thus:

$$S' = \frac{S_{v'} - S_V}{\frac{d^2}{w}} = \frac{S_{1319} - S_{1420}}{{\cdot}1923} = \frac{1762{\cdot}4 - 1251{\cdot}1}{{\cdot}1923};$$

or $S' = ON = 2659$ ft.

But from figure,

$$NB = OB - ON = 3000 - 2659 = 341 \text{ ft.};$$

or the position short of 3000 ft. in which the side of the ship would be hit is 341 ft.

Example 3.—Find the height of the trajectory of the Martini-Henry rifle for a range of 500 yds. at intervals of 100 yds. Weight of bullet $= 480$ grs. $= \frac{480}{7000}$ lbs.* $= {\cdot}06857$ lbs., diameter of bullet $= {\cdot}45$ ins., muzzle velocity $= 1353$ f.s. Method of finding height of trajectory at given distances.

The remaining velocity at 500 yds. $= 1500$ ft. can be found by Table I. thus:

$$S_v = S_{1353} + \frac{({\cdot}45)^2}{{\cdot}06857} \times 1500 = 1584 + 2{\cdot}953 \times 1500 = 6013{\cdot}5;$$

or remaining velocity at 500 yds. = 889 f.s.

Then time over 500 yds. can be found by Table II. thus:

$$2{\cdot}953T = T_{889} - T_{1353} = 5{\cdot}2762 - 1{\cdot}0461 = 4{\cdot}2301;$$

or time over 500 yds. = 1·432 secs.

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* 7000 grs. Troy make 1 lb. Avoirdupois.

The remaining velocity at 100 yds. = 300 ft. can be found by Table I. thus:

$$S_0 = S_{1353} + 2{\cdot}953 \times 300 = 1584 + 885{\cdot}9 = 2469{\cdot}9 \text{ ft.}$$

or velocity at 100 yds. = 1198 f.s.

Then time over 100 yds. can be found by Table II. thus:

$$2{\cdot}953T = T_{1198} - T_{1353} = 1{\cdot}7416 - 1{\cdot}0461 = {\cdot}6955;$$

or time over 100 yds. = ·2356 sec.

Then the height of the trajectory at 100 yds. can be found by substituting in the formula

$$y = \frac{gt}{2}(T - t);$$

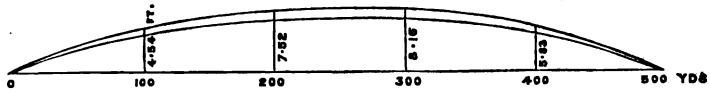
where $T = 1{\cdot}432$ and $t = {\cdot}2356$.

Thus,

$$y_{100} = \frac{32{\cdot}19 \times {\cdot}2356}{2} \times 1{\cdot}1964 = 4{\cdot}537 \text{ ft.}$$

In a similar manner, the height of the trajectory at 200, 300, and 400 yds. may be found.

Trajectory of Martini-Henry and Mauser Rifle at 500 yds.



Martini-Henry.				Mauser.		
Distance.	Velocity.	Time of flight.	Height of trajectory.	Velocity.	Time of flight.	Height of trajectory.
yds.	f.s.	secs.	ft.	f.s.	secs.	ft.
0.....	1353	·0000	0	1526	·0000	0
100.....	1198	·2356	4·54	1312	·2127	3·95
200.....	1075	·5021	7·52	1143	·4578	6·70
300.....	994	·7934	8·16	1023	·7358	7·47
400.....	937	1·1040	5·83	950	1·0400	5·47
500.....	889	1·4320	0	892	1·3670	0
Weight of bullet = 480 grs.; diameter of bullet = ·45 in.				Weight of bullet = 382 grs.; diameter of bullet = ·435 in.		

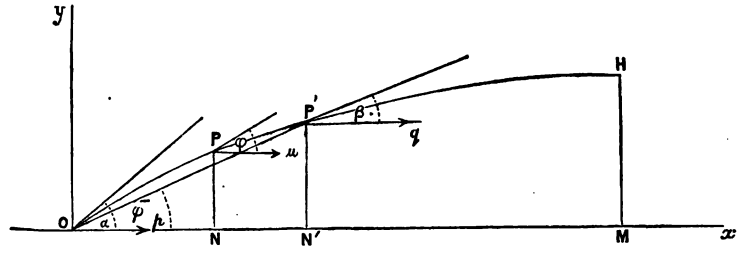
THE MOTION OF A PROJECTILE IN THE AIR WILL NOW BE CONSIDERED.

Motion of a projectile in the air.

It is assumed that the resistance of the air acts along the tangent to the trajectory of the projectile. This assumption is no doubt very nearly correct for flat trajectories, but it cannot be relied upon with certainty for high-angle trajectories. The motion will also be assumed to take place in the plane of the trajectory. This, without doubt, is not quite correct, as it is well known that the projectile performs small gyrations round the mean path of its trajectory.

The same notation will be used as for the trajectory of an unresisted projectile.

v the velocity at any point of the trajectory,
u " horizontal component of v,
ϕ " inclination of the direction of motion to the horizontal line in the plane of the trajectory,
t " time,
x " horizontal distance from the point of projection,
y " the vertical distance.



Suppose O the point of projection, α the angle of departure, OH the ascending path of the projectile, H the vertex of the trajectory, Ox the horizontal line in the plane of the trajectory, Oy the vertical line, P the position of the projectile at any time t, u its horizontal velocity.

It is proposed now to consider the motion of the projectile over a component arc of the trajectory OP', α and β being the inclination of the direction of motion to the horizontal line Ox, at the beginning and end of the arc, p and q being the horizontal velocities—*i.e.*, the values of u—at those points.

Equations of motion.

It has been seen that for certain ranges of velocity the law of retardation may be correctly stated to vary as the cube of the velocity, or to be equal to cv^3.

The equation of motion obtained by resolution in a horizontal direction is, then,

$$\frac{d^2x}{dt^2} = \frac{du}{dt} = -cv^3 \cos\phi. \quad \text{.....(1)}$$

The equation of motion obtained by resolution in the direction of the normal to the trajectory is

$$v\frac{d\phi}{dt} = -g\cos\phi; \quad \text{.....(2)}$$

the sign on the right-hand side of the equation being negative, since ϕ diminishes as t increases.

Hence, dividing (1) by (2),

$$\frac{du}{d\phi} = \frac{cv^4}{g}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

or, since $v = u \sec \phi$,

$$\frac{du}{d\phi} = \frac{cu^4}{g} \sec^4 \phi;$$

$$\text{or} \quad \frac{du}{u^4} = \frac{c}{g} \sec^4 \phi d\phi.$$

Integrating,

$$\int_p^u \frac{du}{u^4} = \frac{c}{g} \int_a^\phi \sec^4 \phi d\phi,$$

$$\text{or} \quad \frac{1}{u^3} - \frac{1}{p^3} = \frac{c}{g} (P_a - P_\phi); \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

where $P_a = 3\int_0^a \sec^4 \phi d\phi = 3 \tan a + \tan^3 a.$

Suppose at the end of the arc OP' the horizontal velocity is q, and β the inclination of the direction of motion to the horizontal line; then, substituting,

$$\frac{1}{q^3} = \frac{1}{p^3} + \frac{c}{g} (P_a - P_\beta). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

The velocity at the vertex of the trajectory.

Suppose u_0 the velocity (horizontal) at the vertex of the trajectory, II, then it follows from the above that

$$\frac{1}{u_0^3} = \frac{1}{p^3} + \frac{c}{g} P_a;$$

$$\text{or} \quad \frac{1}{p^3} = \frac{1}{u_0^3} - \frac{c}{g} P_a. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Substituting this value of $\frac{1}{p^3}$ in equation (4), it follows that

$$\frac{1}{u^3} = \frac{1}{u_0^3} - \frac{c}{g} P_\phi = \frac{1}{u_0^3} \left(1 - \frac{cu_0^3}{g} P_\phi\right).$$

Suppose

$$\frac{cu_0^3}{g} = \gamma, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

then

$$\frac{1}{u^3} = \frac{1}{u_0^3} (1 - \gamma P_\phi);$$

$$\text{or} \quad u = \frac{u_0}{(1 - \gamma P_\phi)^{\frac{1}{3}}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

From equation (2),

$$\frac{dt}{d\phi} = -\frac{v}{g}\sec\phi = -\frac{u\sec^2\phi}{g};$$

and if T represent the time over the arc OP', integrating we have

$$T = \int_{\beta}^{\alpha} \frac{u}{g}\sec^2\phi d\phi;$$

or, substituting the value of u in equation (8),

$$T = \frac{u_0}{g}\int_{\beta}^{\alpha} \frac{\sec^2\phi d\phi}{(1-\gamma P_\phi)^{\frac{1}{4}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Again, $$\frac{dx}{dt} = u,$$

$$\therefore\ x = \int u dt = -\int \frac{u^2\sec^2\phi}{g} d\phi;$$

or if X represent the horizontal distance traversed in the arc OP',

$$X = \frac{u_0^2}{g}\int_{\beta}^{\alpha} \frac{\sec^2\phi}{(1-\gamma P_\phi)^{\frac{1}{4}}} d\phi. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Again, $$\frac{dy}{dt} = u\tan\phi;$$

so that if Y represent the vertical distance traversed in the arc OP',

$$Y = \frac{u_0^2}{g}\int_{\beta}^{\alpha} \frac{\tan\phi\sec^2\phi}{(1-\gamma P_\phi)^{\frac{1}{4}}} d\phi. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Equation (9) gives the time of the projectile over the arc OP', and equations (10) and (11) give the co-ordinates of the projectile at that time.

If the cubic law were correct for all velocities, and consequently the coefficient c constant, the range and time of flight of a projectile could each be correctly determined by this method in two operations—(1) by calculating over the arc from the point of projection to the vertex, (2) by calculating over the arc from the vertex to the point of impact.

But as this coefficient is not constant for all velocities, it becomes necessary to calculate the arcs in smaller parts, and to take the *mean value* of the coefficient for these arcs, and thus to approximate very closely to the correct solution.

Bashforth's method of solution.

The above is the solution which Bashforth has adopted in his valuable treatise on the "Motion of Projectiles,"* in which he has

* "A Mathematical Treatise on the Motion of Projectiles," by F. Bashforth, B.D., published in 1873 by Asher & Co.

given tables for the integrals in equations (9), (10), and (11) corresponding to different values of γ, between certain ranges of angle. Other subsidiary tables are given which materially shorten the calculations. A practical solution of a problem is given at p. 56 of this work.

The value of c, determined by Bashforth, is (*vide* Chap. V., p. 45)

$$c = \frac{d^2}{w} \cdot \frac{K}{(1000)^3},$$

so that equation (6) becomes

$$\left(\frac{1000}{u_0}\right)^3 = \left(\frac{1000}{p}\right)^3 + \frac{d^2}{w}\frac{K}{g} P_a;$$

whence u_0 (the velocity at the vertex) is determined by taking the mean value of K over the arc.

The value of γ is then determined from equation (7) thus:

$$\gamma = \frac{d^2}{w} \cdot \frac{K}{g}\left(\frac{u_0}{1000}\right)^3.$$

The integrals $\int_\beta^\alpha \frac{\sec^2\phi}{(1-\gamma P_\phi)^{\frac{4}{3}}} d\phi$, $\int_\beta^\alpha \frac{\tan\phi \sec^2\phi}{(1-\gamma P_\phi)^{\frac{4}{3}}} d\phi$, and $\int_\beta^\alpha \frac{\sec^2\phi d\phi}{(1-\gamma P_\phi)^{\frac{4}{3}}}$ can then be found from tables at p. 40 *et seq.*, "Motion of Projectiles," and the values of T, X, and Y deduced from equations (9), (10), and (11).

For examples *vide* p. 60 *et seq.*, "Motion of Projectiles," to which treatise the reader is referred for this method of solution.

Niven's method of solution.

Another method of solution has been proposed by Mr. W. D. Niven, of Trinity College, Cambridge, late Professor of Mathematics to the Advanced Class of Royal Artillery Officers at Woolwich, based upon the experiments carried out by Professor Bashforth, and making use of his Tables (Chap. V., p. 55, &c.). It has the advantage of requiring less tabular matter for the approximate solution of problems in practical gunnery.

Using the same notation as before, and putting r for the retardation due to the resistance of the air, the equations of motion obtained by resolution in the horizontal direction and in the direction of the normal to the trajectory in the ascending branch will be, as before,

$$\frac{du}{dt} = -r\cos\phi, \quad \text{.....(i.)}$$

$$v\frac{d\phi}{dt} = -g\cos\phi. \quad \text{.....(ii.)}$$

The principle of this method of solution consists in taking the hori-

zontal velocity u as the independent variable, and expressing the four quantities ϕ, t, x, y in terms of u. Thus, dividing (i.) by (ii.),

$$\frac{du}{vd\phi} = \frac{r}{g},$$

$$\text{or } \frac{d\phi}{du} = \frac{g}{rv};$$

$$\therefore \int_\beta^\alpha d\phi = g\int_q^p \frac{du}{vr},$$

$$\text{or } \alpha - \beta = g\int_q^p \frac{du}{vr}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(a)$$

Again, from (i.),

$$T = \int_q^p \frac{du}{r\cos\phi}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(b)$$

Also, since $\frac{dx}{dt} = u$,

$$dx = udt = -\frac{udu}{r\cos\phi};$$

$$\therefore X = \int_q^p \frac{udu}{r\cos\phi}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(c)$$

Again, since $\frac{dy}{dt} = u\tan\phi$,

$$dy = u\tan\phi\, dt = \frac{u\tan\phi\, du}{r\cos\phi};$$

$$\therefore Y = \int_q^p \frac{u\tan\phi\, du}{r\cos\phi}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(d)$$

The above equations—(a), (b), (c), (d)—are the most general forms of the integrals which give the characteristic quantities in the motion of a projectile. In order to solve them, the form of the quantity r as a function of the velocity must be experimentally determined—*i.e.*, the law of the resistance of the air to the projectile must be known. It has already been shown how this has been accomplished by Mr. Bashforth; but as unfortunately the cubic law only holds for a limited range of velocity, there is no simple formula for r, so that it is necessary to find approximate values of the above integrals. This has already been done when the function of the angle ϕ is outside the integral for the integrals (c), (d) in Table I., and for the integral (b) in Table II. (Chap. V., p. 55, &c.); and Table III. (p. 78) gives, in like manner, the approximate value of the integral in equation (a).

In order to make use of these approximations in the solution of examples, it is necessary to divide the trajectory into arcs as before, and to determine the *mean value* of the quantity ϕ over that arc. For instance, in the figure at p. 69, in the arc OP', the mean value of ϕ, varying from α to β, is the inclination of the chord OP' to the horizontal line. Let this be denoted by $\bar{\phi}$.

The method of finding the mean value of ϕ is too difficult to be given in this treatise; but the reader is referred to a paper by Mr. Niven "On the Calculation of the Trajectories of Shot," in the "Proceedings of the Royal Society," No. 181, 1877.

The inclination $\bar{\phi}$ of the chord OP' is given approximately by

$$\bar{\phi} = \frac{\alpha + \beta}{2} + \tfrac{1}{3}\frac{p - q}{p + q}(\alpha - \beta)$$

for the ascending branch, and

$$\bar{\phi} = \frac{\alpha + \beta}{2} - \tfrac{1}{3}\frac{p - q}{p + q}(\beta - \alpha)$$

for the descending branch.

Without going minutely into the whole treatment of the integrals in equations (a), (b), (c), (d), a general explanation of the method of solution will now be given.

Take, for example, equation (c),

$$X = \int_q^p \frac{u\,du}{r \cos \phi}.$$

Now, r is some function of v—call it $f(v)$; so that equation (c) may be written

$$X = \int_q^p \frac{u\,du}{f(v) \cos \phi} = \int_q^p \frac{u\,du}{f(u \sec \phi) \cos \phi} \text{ (since } v = u \sec \phi).$$

If $\bar{\phi}$ is a mean value of the quantity ϕ over the given arc requiring discussion and determination for this particular integral,

then
$$X = \int_q^p \frac{u\,du}{f(u \sec \bar{\phi}) \cos \bar{\phi}}.$$

Suppose now $u = v^*\cos \bar{\phi}$, then $v = u \sec \bar{\phi}$, and $du = \cos \bar{\phi}\,dv$; so that

$$X = \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{v \cos \bar{\phi}\,dv}{f(v)}.$$

Now, the value of $f(v)$, determined by Bashforth,

$$= cv^3 = \frac{d^2}{w} K \left(\frac{v}{1000}\right)^3;$$

so that

$$X = \cos \bar{\phi} \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{v\,dv}{\frac{d^2}{w} K \left(\frac{v}{1000}\right)^3};$$

$$\text{or } \frac{d^2}{w} X = \cos \bar{\phi} \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{(1000)^3\,dv}{Kv^2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (e)$$

---

* It is scarcely necessary to warn the reader that the v in this substitution is not the same as the v used above for the velocity. It is of course indifferent what symbol may be used in place of the new v, as it always occurs inside of a definite integral.

Similarly,

$$\frac{d^2}{w} Y = \sin \bar{\phi} \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{(1000)^3 dv}{Kv^2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (f)$$

It will be seen that the values of X and Y both depend on the value of the integral

$$\int_v^V \frac{(1000)^3 dv}{Kv^2} = S_v - S_V,$$

given in Table I. (p. 55); so that equations (e) and (f) may be written

$$\frac{d^2}{w} X = \cos \bar{\phi} \, (S_{q \sec \bar{\phi}} - S_{p \sec \bar{\phi}}), \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (A)$$

$$\frac{d^2}{w} Y = \sin \bar{\phi} \, (S_{q \sec \bar{\phi}} - S_{p \sec \bar{\phi}}). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (B)$$

This mode of treatment of the X and Y integrals may be employed in the case of the time integral, equation (b)—viz.,

$$T = \int_q^p \frac{du}{f(v) \cos \phi}.$$

This may be reduced to

$$\frac{d^2}{w} T = \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{(1000)^3 dv}{Kv^3}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (g)$$

$$\text{or } \frac{d^2}{w} T = T_{q \sec \bar{\phi}} - T_{p \sec \bar{\phi}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (C)$$

the value of which is given in Table II. (p. 57).

The mean angle ϕ for the time integral is not, however, quite the same as that for the distance integrals, being approximately

$$\bar{\phi}' = \frac{\alpha + \beta}{2} + \tfrac{1}{6} \frac{p - q}{p + q} (\alpha - \beta)$$

in the ascending branch, and

$$\bar{\phi}' = \frac{\alpha + \beta}{2} - \tfrac{1}{6} \frac{p - q}{p + q} (\beta - \alpha)$$

in the descending branch.

It will be observed that the determination of the distance and time integrals both depend on the value of the quantity $q \sec \bar{\phi}$, of which q is still unknown. This can be found by the consideration of equation (a).

$$\alpha - \beta = g \int_q^p \frac{du}{vr} = g \int_q^p \frac{du}{vf(v)};$$

$$\text{or } \alpha - \beta = g \int_q^p \frac{du}{u \sec \phi f(u \sec \phi)}.$$

As before, suppose ϕ has a mean value $\bar{\phi}$, and suppose $\alpha - \beta$ expressed in degrees $= D$; then

$$D = \frac{180g}{\pi} \int_q^p \frac{du}{u \sec \bar{\phi} f(u \sec \bar{\phi})}.$$

Put $u = v \cos \bar{\phi}$, then the equation becomes

$$D \sec \bar{\phi} = \frac{180g}{\pi} \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{dv}{vf(v)}.$$

But by Bashforth's law,

$$f(v) = \frac{d^2}{w} K \left(\frac{v}{1000}\right)^3;$$

so that
$$\frac{d^2}{w} D = \cos \bar{\phi} \cdot \frac{180g}{\pi} \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{(1000)^3 dv}{Kv^4}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (h)$$

Niven has calculated the value of the integral $\frac{180g}{\pi} (1000)^3 \int_v^{1700} \frac{dv}{Kv^4}$ (using Bashforth's value of K) for values of v down to 900. (*Vide* Table III.) Table III. connects degrees and velocity, and is denoted by D_v.

If this integral is represented by D_v, equation (h) becomes

$$\frac{d^2}{w} D = \cos \bar{\phi} \, (D_{q \sec \bar{\phi}} - D_{p \sec \bar{\phi}}). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (D)$$

There is no means of determining the exact value of the mean angle $\bar{\phi}$ in this case. As, however, sec ϕ varies very slowly for the greater part of the trajectory, it will suffice in this equation to put $\bar{\phi} = \frac{\alpha + \beta}{2}$ for low angles of elevation; but for high angles of elevation $\bar{\phi}$ should be more accurately determined from the formula

$$\tan \bar{\phi} = \frac{\tan \alpha + \tan \beta}{2},$$

by means of a table of natural tangents. With the aid of the D_v Table the value of q sec ϕ can be determined, and thence the value of q.

X, Y, and T can be determined by means of the equations (A), (B), and (C).

In the ascending branch, for the distance integrals, $\frac{1}{3} \frac{p-q}{p+q} (\alpha - \beta)$ must be added to the value of $\bar{\phi}$ so found, and for the time integrals, $\frac{1}{6} \frac{p-q}{p+q} (\alpha - \beta)$ must be added.

In the descending branch, from the value of $\bar{\phi}$ found as above, for

the distance integrals, $\frac{1}{3}\frac{p-q}{p+q}(\beta-\alpha)$ must be subtracted, and for the time integrals, $\frac{1}{6}\frac{p-q}{p+q}(\beta-\alpha)$ must be subtracted.

THE EQUATIONS AND STEPS WILL NOW BE WRITTEN IN THE ORDER OF CALCULATION.

$$\bar{\varphi}_1 = \frac{\alpha+\beta}{2} \text{ for low angles of elevation,}$$

$$\text{or } \tan\bar{\varphi}_1 = \frac{\tan\alpha+\tan\beta}{2} \text{ for high angles of elevation.}$$

If V is the muzzle velocity of the projectile, and α the angle of departure, then

$$p = V\cos\alpha, \text{ and } p\sec\bar{\varphi}_1 = V\cos\alpha\sec\bar{\varphi}_1;$$

then $q\sec\bar{\phi}_1$ can be found from Table III. by equation (D) thus:

$$D_{q\sec\bar{\phi}_1} = \frac{d^2}{w}D\sec\bar{\varphi}_1 + D_{p\sec\bar{\phi}_1};$$

where $D = \alpha-\beta$ in degrees.

The correct value of $\bar{\phi}$ can be found in the ascending branch thus:

$$\bar{\phi} = \bar{\varphi}_1 + \tfrac{1}{3}\frac{p-q}{p+q}(\alpha-\beta);$$

then X can be found from equation (A) thus:

$$X = \frac{w}{d^2}\cos\bar{\varphi}\,(S_{q\sec\bar{\phi}} - S_{p\sec\bar{\phi}}),$$

and Y can be found from equation (B) thus:

$$Y = \frac{w}{d^2}\sin\bar{\varphi}\,(S_{q\sec\bar{\phi}} - S_{p\sec\bar{\phi}}).$$

For the correct determination of the time,

$$\bar{\phi}' = \bar{\phi}_1 + \tfrac{1}{6}\frac{p-q}{p+q}(\alpha-\beta);$$

but as the value of $\bar{\phi}'$ does not occur outside the equation in the time integral, the value ϕ_1 may be taken, and will give accurate results.

Then T can be found from equation (C) thus:

$$T = \frac{w}{d^2}(T_{q\sec\bar{\phi}'} - T_{p\sec\bar{\phi}'}).$$

This process will be repeated for each component arc in the ascending branch. A similar process is carried on for the descending branch; the sum of all the X's in the component arcs gives the range, and the sum of all the T's gives the time of flight.

TABLE III.

Degree and Velocity Table. $\frac{d^2}{v^2} D = D_{\dot{v}} - D_V$.

v.	0	1	2	3	4	5	6	7	8	9
f.s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
90	8·3380	8·2955	8·2529	8·2104	8·1678	8·1253	8·0827	8·0402	7·9976	7·9551
91	7·9125	7·8714	7·8302	7·7890	7·7479	7·7066	7·6654	7·6242	7·5830	7·5418
92	7·5006	7·4627	7·4248	7·3868	7·3489	7·3110	7·2730	7·2351	7·1972	7·1592
93	7·1213	7·0851	7·0489	7·0127	6·9766	6·9404	6·9043	6·8681	6·8319	6·7958
94	6·7596	6·7251	6·6906	6·6561	6·6216	6·5870	6·5525	6·5180	6·4835	6·4490
95	6·4146	6·3819	6·3492	6·3166	6·2839	6·2513	6·2186	6·1860	6·1533	6·1207
96	6·0880	6·0571	6·0263	5·9954	5·9646	5·9338	5·9030	5·8722	5·8413	5·8105
97	5·7797	5·7507	5·7217	5·6927	5·6637	5·6347	5·6057	5·5767	5·5477	5·5187
98	5·4898	5·4625	5·4352	5·4079	5·3806	5·3533	5·3260	5·2987	5·2715	5·2442
99	5·2169	5·1914	5·1660	5·1405	5·1151	5·0896	5·0641	5·0387	5·0132	4·9878
100	4·9623	4·9386	4·9149	4·8912	4·8675	4·8437	4·8200	4·7963	4·7726	4·7489
101	4·7252	4·7032	4·6812	4·6591	4·6371	4·6151	4·5931	4·5711	4·5490	4·5270
102	4·5050	4·4847	4·4643	4·4440	4·4236	4·4033	4·3830	4·3626	4·3423	4·3219
103	4·3016	4·2829	4·2642	4·2456	4·2269	4·2082	4·1895	4·1708	4·1522	4·1335
104	4·1148	4·0977	4·0806	4·0635	4·0464	4·0293	4·0122	3·9951	3·9780	3·9609
105	3·9439	3·9282	3·9125	3·8968	3·8811	3·8655	3·8498	3·8341	3·8184	3·8027
106	3·7871	3·7726	3·7581	3·7436	3·7292	3·7147	3·7002	3·6857	3·6712	3·6567
107	3·6422	3·6287	3·6151	3·6016	3·5881	3·5746	3·5610	3·5475	3·5340	3·5204
108	3·5069	3·4941	3·4814	3·4686	3·4558	3·4431	3·4303	3·4175	3·4047	3·3920
109	3·3792	3·3670	3·3549	3·3427	3·3306	3·3184	3·3062	3·2941	3·2819	3·2698
110	3·2576	3·2460	3·2344	3·2228	3·2111	3·1995	3·1879	3·1762	3·1646	3·1529
111	3·1413	3·1301	3·1190	3·1078	3·0966	3·0855	3·0743	3·0631	3·0519	3·0408
112	3·0296	3·0189	3·0081	2·9974	2·9867	2·9760	2·9652	2·9545	2·9438	2·9330
113	2·9223	2·9120	2·9017	2·8913	2·8810	2·8707	2·8604	2·8501	2·8397	2·8294
114	2·8191	2·8092	2·7992	2·7893	2·7794	2·7695	2·7595	2·7496	2·7397	2·7297
115	2·7198	2·7102	2·7007	2·6916	2·6816	2·6721	2·6625	2·6530	2·6435	2·6339
116	2·6244	2·6152	2·6059	2·5967	2·5874	2·5782	2·5689	2·5597	2·5504	2·5412
117	2·5319	2·5230	2·5141	2·5052	2·4963	2·4874	2·4785	2·4696	2·4607	2·4518
118	2·4429	2·4343	2·4257	2·4171	2·4085	2·3999	2·3913	2·3827	2·3741	2·3655
119	2·3570	2·3486	2·3403	2·3320	2·3237	2·3154	2·3071	2·2988	2·2905	2·2822
120	2·2739	2·2659	2·2578	2·2498	2·2418	2·2338	2·2257	2·2177	2·2097	2·2016
121	2·1936	2·1858	2·1781	2·1703	2·1625	2·1548	2·1470	2·1392	2·1314	2·1237
122	2·1159	2·1084	2·1008	2·0933	2·0858	2·0783	2·0707	2·0632	2·0557	2·0481
123	2·0406	2·0333	2·0260	2·0188	2·0115	2·0042	1·9969	1·9896	1·9824	1·9751
124	1·9678	1·9607	1·9537	1·9466	1·9396	1·9325	1·9254	1·9184	1·9113	1·9043
125	1·8972	1·8903	1·8834	1·8766	1·8697	1·8629	1·8561	1·8492	1·8424	1·8355
126	1·8287	1·8221	1·8154	1·8088	1·8022	1·7956	1·7889	1·7823	1·7757	1·7690
127	1·7625	1·7560	1·7496	1·7432	1·7367	1·7303	1·7239	1·7174	1·7110	1·7045
128	1·6981	1·6919	1·6856	1·6794	1·6731	1·6669	1·6606	1·6544	1·6481	1·6419

TABLE III.—(*Continued.*)

v.	0	1	2	3	4	5	6	7	8	9
f.s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
129	1·6356	1·6295	1·6234	1·6174	1·6113	1·6052	1·5991	1·5931	1·5870	1·5810
130	1·5749	1·5689	1·5630	1·5571	1·5512	1·5453	1·5394	1·5334	1·5275	1·5216
131	1·5157	1·5099	1·5042	1·4984	1·4927	1·4870	1·4813	1·4756	1·4698	1·4641
132	1·4584	1·4528	1·4472	1·4417	1·4361	1·4305	1·4249	1·4193	1·4138	1·4082
133	1·4026	1·3972	1·3918	1·3863	1.3809	1·3755	1·3701	1·3646	1·3592	1·3538
134	1·3484	1·3431	1·3378	1·3325	1·3272	1·3220	1·3167	1·3114	1·3061	1·3008
135	1·2955	1·2904	1·2852	1·2801	1·2749	1·2698	1·2646	1·2595	1·2543	1·2492
136	1·2440	1·2390	1·2340	1·2289	1·2239	1·2189	1·2139	1·2089	1·2038	1·1988
137	1·1938	1·1889	1·1840	1·1791	1·1742	1·1693	1·1644	1·1595	1·1546	1·1497
138	1·1448	1·1400	1·1352	1·1305	1·1257	1·1209	1·1161	1·1113	1·1066	1·1018
139	1·0970	1·0923	1·0876	1·0829	1·0783	1·0736	1·0689	1·0643	1·0596	1·0549
140	1·0503	1·0457	1·0412	1·0366	1·0321	1·0275	1·0229	1·0184	1·0138	1·0093
141	1·0047	1·0002	·9958	·9913	·9869	·9824	·9779	·9735	·9690	·9646
142	·9601	·9557	·9514	·9470	·9427	·9383	·9339	·9296	·9252	·9209
143	·9165	·9122	·9079	·9036	·8994	·8951	·8908	·8866	·8823	·8780
144	·8737	·8695	·8653	·8612	·8570	·8528	·8486	·8444	·8402	·8361
145	·8319	·8278	·8237	·8196	·8155	·8114	·8073	·8032	·7991	·7950
146	·7910	·7870	·7830	·7790	·7750	·7710	·7669	·7629	·7589	·7549
147	·7509	·7470	·7430	·7391	·7350	·7312	·7273	·7234	·7194	·7155
148	·7115	·7077	·7038	·6999	·6961	·6922	·6884	·6845	·6806	·6768
149	·6729	·6691	·6654	·6616	·6578	·6540	·6502	·6465	·6427	·6389
150	·6351	·6314	·6277	·6239	·6202	·6165	·6128	·6091	·6053	·6016
151	·5979	·5943	·5906	·5870	·5833	·5797	·5760	·5724	·5687	·5651
152	·5614	·5578	·5542	·5506	·5470	·5435	·5399	·5363	·5327	·5291
153	·5255	·5220	·5185	·5149	·5114	·5079	·5044	·5009	·4973	·4938
154	·4903	·4868	·4834	·4799	·4765	·4730	·4695	·4661	·4626	·4592
155	·4557	·4523	·4489	·4455	·4421	·4387	·4352	·4318	·4284	·4250
156	·4216	·4183	·4149	·4116	·4082	·4049	·4015	·3982	·3948	·3915
157	·3881	·3848	·4815	·3782	·3749	·3716	·3684	·3651	·3618	·3585
158	·3552	·3520	·3487	·3455	·3422	·3390	·3357	·3325	·3292	·3260
159	·3227	·3195	·3163	·3132	·3100	·3068	·3036	·3004	·2973	·2941
160	·2909	·2877	·2846	·2814	·2783	·2751	·2719	·2688	·2656	·2625
161	·2593	·2562	·2531	·2500	·2469	·2439	·2408	·2377	·2346	·2315
162	·2284	·2254	·2223	·2193	·2162	·2132	·2102	·2071	·2041	·2010
163	·1980	·1950	·1920	·1890	·1860	·1831	·1801	·1771	·1741	·1711
164	·1681	·1652	·1622	·1593	·1563	·1534	·1505	·1475	·1446	·1416
165	·1387	·1358	·1329	·1300	·1271	·1243	·1214	·1185	·1156	·1127
166	·1098	·1070	·1042	·1013	·0985	·0957	·0929	·0900	·0872	·0844
167	·0816	·0788	·0760	·0732	·0705	·0677	·0649	·0621	·0594	·0566
168	·0538	·0511	·0484	·0456	·0429	·0402	·0375	·0347	·0320	·0293
169	·0266	·0239	·0213	·0186	·0159	·0133	·0106	·0080	·0053	·0027

The notation of Table III. and the method of using it is similar to that of Tables I. and II., explained in Chap. V. (p. 51).

Practical examples.

EXAMPLE (1).—What is the range and time of flight of a 12-lb. shot fired with a muzzle velocity of 1680 f.s. at 3° of quadrant elevation; the jump being taken at 20′, and the diameter of the shot 3 ins.?

Here angle of departure, α, $= 3° 20'$,

$$\frac{d^2}{w} = \tfrac{3}{4}.$$

The whole ascending branch of the trajectory may be taken, and at the vertex

$$\beta = 0;$$

hence

$$D = \alpha - \beta = 3° 20'.$$

In applying the formula for small angles of departure,

$$D_{q \sec \bar{\phi}} = D_{p \sec \bar{\phi}} + \frac{d^2}{w} D \sec \bar{\phi}.$$

$\sec \bar{\phi}$ may be neglected, and $p \sec \bar{\phi}$ may be taken equal to V, the muzzle velocity, and $q \sec \bar{\phi} = v$, the remaining velocity at the end of the arc under consideration; so that the above formula becomes

$$D_v = D_V + \frac{d^2}{w} D = D_{1680} + \tfrac{3}{4} \times 3{\cdot}333$$
$$= {\cdot}0538 + 2{\cdot}5 = 2{\cdot}5538;$$
$$\text{or } v = 1168.$$

The formula for computing $\bar{\phi}$ is in this case, where $\beta = 0$,

$$\bar{\phi} = \frac{\alpha}{2} + \frac{p - q}{p + q} \frac{\alpha}{3};$$

and since $p = V$ and $q = v$ nearly,

$$\bar{\phi} = 1° 40' + \frac{1680 - 1168}{1680 + 1168} \times \frac{300'}{3} = 1° 40' + 12' = 1° 52'.$$

The formula for computing X is

$$\frac{d^2}{w} X = \cos \bar{\phi} \, (S_{q \sec \bar{\phi}} - S_{p \sec \bar{\phi}});$$

which may be written in this case

$$\frac{d^2}{w} X = \cos \bar{\phi} \, (S_v - S_V);$$
$$\text{or } X = \frac{4 \times \cos 1° 52'}{3} (S_{1168} - S_{1680}) \qquad \text{since } S_{1168} = 2665, \; S_{1680} = 83, \; 2582$$
$$= 3443 \cos 1° 52'$$
$$= 3441 \text{ ft.}$$

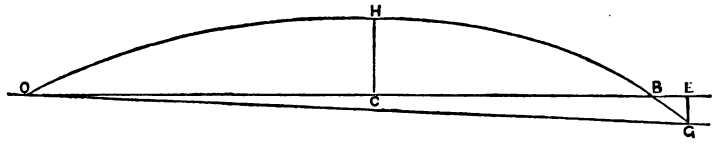
Similarly,

$$Y = \cdot 3443 \sin 1^\circ 52'$$
$$= 112 \cdot 2 \text{ ft.}$$

Similarly,

$$T = \tfrac{4}{3}(T_{1168} - T_{1680})$$
$$= 2 \cdot 48 \text{ secs.}$$

since $T_{1168} = 1 \cdot 9081$
$T_{1680} = \cdot 0493$
$1 \cdot 8588$



To explain: the arc OH (where H is the vertex of the trajectory) has been computed, and

$$OC = X = 3441 \text{ ft.},$$
$$HC = Y = 112 \text{ ft.},$$

or the height which the shot would have to fall from the vertex to reach a horizontal line through the muzzle of the gun. But if the range is measured to a lower level, G, the distance EG must be added to the height the shot has to fall—suppose in this case 15 ft.; then the height the shot has to fall from the vertex is $112 + 15 = 127$ ft.

In computing the descending branch, it is necessary to decide upon some angle at the end of the arc and as near as possible to the actual angle of descent. In this case compute the descending branch from 0° to 5°.

Then, proceeding as before, since $D = 5^\circ$,

$$D_v = D_v + \frac{d^2}{w} D = D_{1168} + \tfrac{3}{4} \times 5 = 6 \cdot 3;$$

since $D_{1168} = 2 \cdot 550$
$+ 3 \cdot 75$
$6 \cdot 300$

$$\therefore\ v' = 953 \cdot 5 \text{ f.s.},$$

and

$$\varphi' = 2^\circ 30' - \frac{1168 - 953}{1168 + 953} \times \frac{300'}{3}$$
$$= 2^\circ 30' - 10' = 2^\circ 20';$$

then

$$X' = \tfrac{4}{3} \cos 2^\circ 20' (S_{953 \cdot 5} - S_{1168})$$
$$= 2917 \cos 2^\circ 20' = 2914 \text{ ft.},$$
$$Y' = 2917 \sin 2^\circ 20' = 119 \text{ ft. nearly.}$$

since $S_{953 \cdot 5} = 4853$
$S_{1168} = 2665$
2188
4
$3)8752$
2917

i.e., there is yet $127 - 119$ ft. $= 8$ ft. to fall.

The little extra distance due to a fall of 8 ft. may be approximated to by

$$X'' = 8 \cot 5° = 8 \times 11{\cdot}43 = 91{\cdot}4 \text{ ft.}$$

Now,

$$\begin{aligned} X &= 3441 \\ X' &= 2914 \\ X'' &= 91 \end{aligned}$$

or total range = 6446 ft. = 2148·6 yds.

Again,

$$T' = \tfrac{4}{3}\,(T_{953{\cdot}5} - T_{1168})$$

$$= 2{\cdot}81 \text{ secs.}$$

$$\begin{aligned} T_{953{\cdot}5} &= 4{\cdot}0137 \\ T_{1168} &= 1{\cdot}9081 \\ & \overline{2{\cdot}1056} \\ & 4 \\ & \overline{3)8{\cdot}4224} \\ & \overline{2{\cdot}8074} \end{aligned}$$

The little extra time taken to travel over $X'' = 91$ ft. may be approximated to by dividing the distance by the velocity at end of the arc of 5°—*i.e.*, by $v' = 953$ f.s.; or

$$T'' = \frac{91}{953} = {\cdot}095.$$

Now,

$$\begin{aligned} T &= 2{\cdot}48 \\ T' &= 2{\cdot}81 \\ T'' &= {\cdot}09 \end{aligned}$$

or total time of flight = 5·38 secs.

In practice with this gun at Shoeburyness under similar conditions, the mean range of 10 rounds was found to be 2128 yds.—the maximum being 2161 yds. and the minimum 2097. The mean time of flight was observed to be 5·5 secs.

AN EXAMPLE OF THE APPROXIMATE SOLUTION OF HIGH-ANGLE TRAJECTORIES WILL NOW BE GIVEN.

EXAMPLE (2).—The 8-in. M.L. howitzer is fired at an angle of quadrant elevation of 30°, and jumps 1°. What will be the range and time of flight? Muzzle velocity = 745 f.s., weight of shell = 180 lbs., diameter of shell = 7·92 ins.

The most correct way of solution would be to divide the ascending branch into three or more component arcs, and compute over each arc; and in the same way with the descending branch. But this would

make the calculation too long for ordinary purposes. The ascending branch will be computed in one step.

Here $\alpha = 31°,\ \beta = 0,\ D = 31°,$

$$\log \frac{d^2}{w} = 9{\cdot}5422,\ V = 745,$$

$$\tan \bar{\phi}_1 = \frac{\tan 31°}{2} = \frac{{\cdot}60086}{2} = {\cdot}30043;$$

$$\therefore\ \bar{\phi}_1 = 16° 43',$$

$$\text{and } p \sec \bar{\phi}_1 = 745 \cos 31° \sec 16° 43' = 667 \text{ f.s.},$$

$$\text{also } p = 638{\cdot}6.$$

$q \sec \bar{\phi}_1$ may be found by substituting in equation (i.) [note],

$$\left(\frac{1000}{q \sec \bar{\phi}_1}\right)^3 = \frac{\frac{d^2}{w} \times 31 \times \sec 16° 43'}{9{\cdot}54655} + \left(\frac{1000}{667}\right)^3$$

$$= 1{\cdot}182 + 3{\cdot}370 = 4{\cdot}552,$$

$$\text{and } q \sec \bar{\phi}_1 = 603{\cdot}4 \text{ f.s.};$$

$$\text{or } q = 577{\cdot}8.$$

The correct value of $\bar{\phi}$, then, is

$$\bar{\phi} = \bar{\phi}_1 + \frac{p - q}{p + q} \cdot \frac{\alpha}{3}$$

$$= 16° 43' + \frac{639 - 578}{639 + 578} \times \frac{31 \times 60}{3}$$

$$= 16° 43' + \frac{61 \times 620}{1217} = 16° 43' + 31' = 17° 14',$$

$$\frac{d^2}{w} X = \cos \bar{\phi} \left(S_{q \sec \bar{\phi}} - S_{p \sec \bar{\phi}}\right)$$

$$= \cos 17° 14' \left(S_{603{\cdot}4} - S_{667}\right);$$

$$S_{603{\cdot}4} = 14282$$
$$S_{667} = 11828$$
$$2454$$

$$\text{or } X = 6726 \text{ ft.},$$

$$Y = X \tan 17° 14' = 2086 \text{ ft.},$$

$$T = \frac{w}{d^2}\left(T_{603{\cdot}4} - T_{667}\right)$$

$$= 11{\cdot}06 \text{ secs.}$$

$$T_{603{\cdot}4} = 16{\cdot}777$$
$$T_{667} = 12{\cdot}904$$
$$3{\cdot}873.$$

N.B.—As Table III. is not calculated below 900 f.s., it becomes necessary to make the calculation independently.

$$\frac{d^2}{w} D \sec \bar{\phi}_1 = D_{q \sec \bar{\phi}_1} - D_{p \sec \bar{\phi}_1} = \frac{180g}{\pi} \cdot \frac{(1000)^3}{64{\cdot}4} \int_{q \sec \bar{\phi}_1}^{p \sec \bar{\phi}_1} \frac{dv}{v^4}$$

(since $K = 64{\cdot}4$, and is *supposed* constant below 900 f.s.)

$$= \frac{180 \times 32{\cdot}2}{3\pi} \times \frac{1}{64{\cdot}4} \left\{\left(\frac{1000}{q \sec \bar{\phi}_1}\right)^3 - \left(\frac{1000}{p \sec \bar{\phi}_1}\right)^3\right\}$$

$$= 9{\cdot}54655 \times \left\{\left(\frac{1000}{q \sec \bar{\phi}_1}\right)^3 - \left(\frac{1000}{p \sec \bar{\phi}_1}\right)^3\right\};$$

$$\text{or } \left(\frac{1000}{q \sec \bar{\phi}_1}\right)^3 = \frac{\frac{d^2}{w} D \sec \bar{\phi}_1}{9{\cdot}54655} + \left(\frac{1000}{p \sec \bar{\phi}_1}\right)^3 \quad \text{.....(i.)}$$

In the descending branch, compute from 0 to 34·2°; then

$$\beta = 34{\cdot}2^\circ,\ D = 34{\cdot}2^\circ,$$

$$\tan \bar{\varphi}_1 = \frac{\tan 34^\circ\ 12'}{2} = {\cdot}3398\,;$$

$$\text{or } \bar{\varphi}_1 = 18^\circ\ 46',$$

$$p = 577{\cdot}8, \text{ and } p \sec \bar{\varphi}_1 = 610{\cdot}3,$$

$$\left(\frac{1000}{q \sec \bar{\varphi}_1}\right)^3 = \frac{\frac{d^2}{w} \times 34{\cdot}2 \times \sec 18^\circ\ 46'}{9{\cdot}5466} + \left(\frac{1000}{610{\cdot}3}\right)^3$$

$$= 1{\cdot}319 + 4{\cdot}400 = 5{\cdot}719,$$

$$\text{or } q \sec \bar{\varphi}_1 = 559{\cdot}2 \text{ f.s., and } q = 529{\cdot}5\,;$$

$$\text{then } \bar{\varphi} = 18^\circ\ 46' - \frac{578 - 530}{578 + 530} \times \frac{34^\circ\ 12'}{3}$$

$$= 18^\circ\ 46' - 30' = 18^\circ\ 16',$$

$$\frac{d^2}{w} X' = \cos 18^\circ\ 16'\ (S_{559{\cdot}2} - S_{610{\cdot}3})\,; \qquad \begin{array}{l} S_{559{\cdot}2} = 16316 \\ S_{610{\cdot}3} = 13991 \\ \hline 2325 \end{array}$$

$$\text{or } X' = 6334 \text{ ft.},$$

$$Y' = X' \tan 18^\circ\ 16' = 2091 \text{ ft.},$$

$$T' = \frac{w}{d^2}(T_{559{\cdot}2} - T_{610{\cdot}3}) = 3{\cdot}984 \times \frac{w}{d^2} \qquad \begin{array}{l} T_{559{\cdot}2} = 20{\cdot}281 \\ T_{610{\cdot}3} = 16{\cdot}297 \\ \hline 3{\cdot}984 \end{array}$$

$$= 11{\cdot}43 \text{ secs.},$$

$$\text{range} = X + X' = 6726 + 6334$$

$$= 13060 \text{ ft.}$$

$$= 4353 \text{ yds.},$$

$$\text{time of flight} = T + T' = 11{\cdot}06 + 11{\cdot}43 = 22{\cdot}49 \text{ secs.},$$

$$\text{and } Y - Y' = 2086 - 2091 = -5 \text{ ft.}$$

or the projectile strikes 5 ft. below the horizontal line.

From above,

$$q = 529{\cdot}5,$$

so that

$$\text{striking velocity} = 529{\cdot}5 \sec 34^\circ\ 12' = 640{\cdot}3 \text{ f.s.}$$

The same example worked out more carefully by Bashforth's method of solution gives

$$\text{range} = 13197 \text{ ft.} = 4399 \text{ yds.}$$

$$\text{time of flight} = 22{\cdot}87 \text{ secs.}$$

$$\text{and striking velocity} = 635 \text{ f.s.}$$

When the experiments for determining the coefficients for velocities below 900 f.s. are completed, and Tables I., II., and III. extended in accordance with the results observed, the solution of high-angle trajectories will be approached with greater confidence.

CHAPTER VII.

Drift of Elongated Projectiles.

Rotation round the Longer Axis of a Projectile.—Method of giving Rotation in Service Ordnance.—Circumstances which Determine the Velocity of Rotation required to keep a Projectile Steady in Flight.—Effect of Non-Centring.—Effect of the Resistance of the Air.—Position of the Centre of Gravity.—Length of the Projectile.—Distribution of the Mass of the Projectile.—The Spiral Motion of an Elongated Projectile.—Illustrated by means of a Gyroscope.—Drift of an Ogival-Headed Projectile.—Drift of a Flat-Headed Projectile.—Varying Conditions which affect the Drift.—Practical Conclusions.

Rotation round the longer axis of a projectile.

It has already been stated (Chap. I. p. 1) that sufficient rotation is usually given to an elongated projectile round its longer axis to keep it *as nearly as possible* point first, or, in other words, in the direction of the tangent to the trajectory, during flight.

It is well known that if a service elongated projectile were fired with no rotation round its longer axis, it would oscillate about its shorter axis through the centre of gravity, and tend to set its longer axis at right angles to the direction of motion.

Method of giving rotation in service ordnance.

Rotation is given in service ordnance in three ways:—

(1) By means of a soft metal *coating* on the cylindrical surface of the projectile (slightly larger than the diameter of the bore) being forced into the spiral grooves of the bore; as in the Armstrong B.L. guns.

(2) By means of *studs* on the projectiles fitting the spiral grooves of the bore; as in the French and Woolwich systems of rifling.

(3) By means of a metal *disc* attached to the base of the projectile, and expanded into the spiral grooves of the bore by the impulse on firing; as in the more recent ordnance with polygroove rifling.

Circumstances which determine the velocity of rotation required to keep a projectile steady in flight.

The velocity of rotation required to keep a projectile steady in flight depends—

(1) On the centring, or otherwise, of the projectile as it leaves the bore.

(2) On the effect of the resistance of the air which the projectile meets with, tending to give it rotation round its shorter axis.

(3) On the position of the centre of gravity of the projectile.

(4) On the length of the projectile in calibres.

(5) On the distribution of the mass of the projectile.

Effect of non-centring. If the projectile is centred as it leaves the bore, there will be no force tending to cause rotation round its shorter axis until gravity deflects the trajectory from the line of departure, and in this case the force is gradually applied, and in a constant direction, producing similar steady gyration; but if the projectile be not centred, there will be at once on leaving the bore a force causing rotation round its shorter axis, acting in an uncertain direction, and producing dissimilar unsteady gyration.* In order to keep the semi-amplitude of the gyration small, it is necessary to give a higher velocity of rotation than would have been required if the projectile had been centred; so that the axis of the projectile may be kept as nearly as possible in the direction of motion of the centre of gravity of the projectile, or, in other words, tangential to the trajectory. If the velocity of rotation is not sufficient, the semi-amplitude of the gyration will increase, and the projectile becomes unsteady, and wabbles in flight. In extreme cases the projectile will turn over; as, for instance, in long shell when fired with a low velocity.

Effect of the resistance of the air. The effect of the resistance of the air, and the point of application of that resistance on projectiles of different shapes, will next be considered.

Fig. 1.

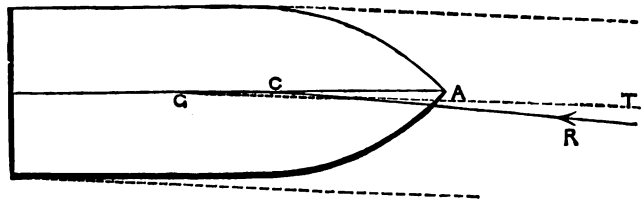


Fig. 1 represents a vertical projection of an ogival-headed projectile, *G* the position of its centre of gravity in its longer axis, *GA*. The direction of motion of the projectile is represented by *GT*, being a tangent to the trajectory, which has dipped from the line of departure, which if the projectile is centred is coincident at the muzzle with the axis of the projectile, *GA*.

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* Gyration must be understood to mean the rotation of the axis of the projectile itself round the trajectory described by its centre of gravity. The semi-amplitude of the gyration is the angle which the axis of the projectile makes with the tangent to the trajectory at that point.

The front part of the projectile is, then, meeting the air in directions parallel to *GT*. The resultant of the resistance of the air opposed to this motion does not usually (owing to the form of head) act in exactly the opposite direction, but in a direction inclined to it, as *RC*. The point *C*, where the direction of the resultant of the resistance intersects the axis of the projectile, is called the "centre of resistance." The resultant *R* acts in the direction *RC* in the plane passing through the axis of the projectile and the tangent to the trajectory.

The resultant, *R*, acting at *C* may be replaced by an equivalent force, *R*, acting at *G*, the centre of gravity of the projectile, and parallel to *RC*, and a couple whose moment is proportional to $R \times GC$. The tendency of the couple, as drawn in Fig. 1 (which represents the case of an ogival-headed projectile) is to *raise the point of the projectile—i.e.*, to give the projectile rotation round its shorter axis passing through its centre of gravity, and perpendicular to its longer axis.

The combination of this rotation and the rotation impressed on the projectile round its longer axis by the spiral grooves in the bore, gives rise to the gyration of the axis of the projectile round a tangent to its trajectory. If the moment of this couple is increased from any cause, the velocity of rotation of the projectile must also be proportionally increased, in order to keep the axis of the projectile as close to the trajectory as possible; or, in other words, to keep the semi-amplitude of the gyration within moderate limits.

The magnitude of the resultant resistance for a projectile of given diameter depends principally on the velocity and form of head of the projectile. Hence, when a projectile is fired with a high muzzle velocity, since the moment about the centre of gravity increases with the magnitude of the resultant resistance, a higher velocity of rotation is required to keep it steady under similar circumstances.

Flat-headed projectiles, for the same reason, require a higher velocity of rotation than ogival-headed projectiles.* In experiments made by Colonel Owen, R.A., "on the derivation of elongated projectiles," it was found "that the velocity of rotation given by the rifled B.L. 40-pr. was not sufficiently high to keep a flat-headed shot steady during flight, except at very short ranges."†

Position of the centre of gravity.

The moment of the couple tending to produce rotation round the shorter axis, depends on the relative position of the centre of gravity and the centre of resistance—*i.e.*, on the distance *GC*, in Fig. 1. The greater this distance is, the greater will be the tendency to upset the projectile, and the greater must be the *velocity of rotation*, in order to keep the projectile steady in flight. If the centre of gravity of the projectile were purposely brought towards the base, the tendency of the projectile would be to fly base foremost, unless it were counteracted by a much greater velocity of rotation than would be necessary under

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* The coefficient of resistance of flat-headed shot is much greater than for the ogival head (*vide* p. 45). For flat-headed shot, $K = 139{\cdot}6$.

† *Vide* Owen's "Modern Artillery," p. 245.

service conditions. If, on the contrary, the centre of gravity is towards the head, this distance is smaller, and a lower velocity of rotation suffices to keep the projectile steady in flight; for instance, in the Snider bullet the centre of gravity is more towards the head than it would be if the bullet were solid, and the very low velocity of rotation imparted to it ensures tolerable accuracy of fire.

Length of the projectile.

Generally speaking, the longer a projectile is in calibres for a given weight, the greater will be the velocity of rotation required to keep its axis close to the trajectory; because the resultant of the resistance usually acts with a greater leverage—*i.e.*, the moment of the couple tending to produce rotation is increased as the length of the projectile is increased. But it is quite possible, with some shapes of head and positions of centre of gravity, for the moment of the couple to be less with a longer projectile than with a shorter one of the same weight. In that case a greater velocity of rotation may be dispensed with. The principle to be remembered in considering the velocity of rotation to be given to an elongated projectile in order to keep it steady in flight is this—that the *velocity of rotation must be increased proportionately to the moment of the resultant resistance about the centre of gravity tending to rotate the projectile round its shorter axis.*

Distribution of the mass of the projectile.

The rate of loss of velocity of rotation owing to the resistance of the air depends on the distribution of the mass of the projectile. The velocity of rotation is maintained longer in the case of an elongated shell than in that of an elongated shot of given weight. Hence it is found in practice that an elongated shell is steadier in flight than an elongated shot of the same weight; the reason being that the radius of gyration is longer in the shell, and for a given velocity of rotation the energy of rotation is greater. (*Vide* p. 15.)

The spiral motion of an elongated projectile.

The direction of the rotation given to all service elongated projectiles is "right-handed"—*i.e.*, the upper part turns from left to right, with reference to an observer placed behind the gun. If the projectile is accurately centred, there will be no tendency to gyrate until gravity deflects the trajectory from the line of departure, and the direction of motion of the projectile ceases to be coincident with its axis. When this is the case, the resultant of the resistance of the air acts as in Fig. 1, tending (as explained before) to raise the point of the projectile, and to give it rotation round the shorter axis through the centre of gravity.* The combination of these rotations, in obedience to a well-known law in Mechanics, causes the axis of the projectile to gyrate in a right-handed direction; and the resistance of the air acting on the left side of the projectile drives the projectile itself bodily towards the right.

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* *Vide* a paper by Major Morgan, R.A., on the "Rifling for Heavy Guns," read at the United Service Institution on May 19th, 1873; also a paper by General Mayevski in the "Revue de Technologie Militaire," Tome V. p. 13.

The number of gyrations depends upon the velocity of the projectile: a projectile fired with a high velocity will make several gyrations, while a projectile fired with only a low velocity and at a high angle of elevation would only make a part of one gyration.

Fig. 2.

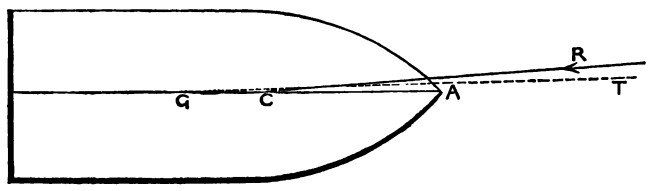

Fig. 2 represents a horizontal projection of an ogival-headed projectile at the end of the first quarter-revolution of its axis round the tangent to the trajectory—*i.e.*, during the first quarter of gyration. *G* represents the position of the centre of gravity in its longer axis, *GA*. The direction of motion of the projectile in the horizontal plane is represented by *GT*. The axis, *GA*, of the projectile is deflected to the right of this line. *RC* represents the direction of the resultant of the resistance of the air. The resolved part of this resultant resistance perpendicular to *GT causes the projectile itself to drift bodily to the right;* it also tends to rotate the projectile round its shorter axis through the centre of gravity, *G*. The combination of this rotation with the rotation impressed on the projectile round its longer axis causes the axis of the projectile to dip, and thus to commence the second quarter-gyration. Fig. 2, then, represents a *vertical* projection at the end of the second quarter of gyration.

The resolved part of the resultant resistance perpendicular to *GT* now drives the projectile bodily downwards, and at the same time tends to produce rotation by deflecting the axis of the projectile from the direction of motion. This rotation, combined with the rotation of the projectile, causes the axis of the projectile to deflect to the *left* (Fig. 1 may now be taken as the horizontal projection under these conditions); and the resolved part of the resultant in the third quarter of gyration drives the projectile bodily to the *left*, and tends to cause rotation, which, combined with the rotation of the projectile, causes the axis of the projectile to rise, and thus to commence the fourth quarter of gyration.

Illustrated by means of a gyroscope.

A gyroscope, with an elongated shot with a rounded head, serves to explain this motion. If the elongated shot is made to rotate in the direction of the hands of a clock—*i.e.*, with a right-handed rotation—and a pressure applied by means of a pencil-point on the head of the shot below its axis, then the axis of the shot will deflect to the right, with reference to an observer looking at it from the rear. If now the pressure is applied on the left front, the axis of the shot will dip; if

the pressure is applied on the head *above* the axis, the axis of the shot will deflect to the *left;* and finally, to complete the gyration, if the pressure is applied on the right front, the axis of the shot will rise. The pressure of the pencil-point corresponds with the direction of the resultant of the resistance of the air in the four quarters of gyration.

Drift of an ogival-headed projectile.

It will be seen that when the projectile is in motion and in the first and second quarters of gyration, it is driven to the *right,* and in the third and fourth quarters of gyration the projectile is driven to the *left,* by the resistance of the air; but as the time taken in the first and second quarters of gyration is longer than that taken in the third and fourth quarters,* the drift or derivation of the projectile is on the whole to the *right* of the plane of departure.

If *left*-handed rotation is impressed on the shot in the gyroscope, gyration under similar conditions will take place in the opposite direction, the axis of the shot first deflecting to the left.

If a projectile has left-handed rotation impressed upon it, gyration takes place in the opposite direction, and the drift will be in the first and second quarters of gyration to the left of the plane of departure, and in the third and fourth to the right; but as the time taken in the first and second quarters of gyration is longer than in the third and fourth quarters of gyration, the drift on the whole will be to the *left.*

In the above description of the spiral motion of an elongated ogival-headed projectile, the projectile has been supposed to be centred; but if this should not be the case gyration commences at once, and in an uncertain direction, and the axis of the projectile will be deflected at once, according to the position of the point on which the resultant of the resistance of the air acts on the projectile, and the projectile itself driven bodily in the same direction. Hence the shooting of guns in which the centring of the projectile is not secured cannot be relied upon for such accuracy as those guns in which the projectile is centred, because the drift cannot be so accurately allowed for.

Since the use of copper gas-checks in the 12·5-in. M.L. gun of 38 tons, the projectile has been centred, and the mean drift to the right for a given range has increased, and the shooting has been more accurate.

Drift of a flat-headed projectile.

The drift of flat-headed projectiles is not so marked as that of ogival-headed projectiles, and owing to the higher velocity of rotation required to keep flat-headed projectiles steady in flight, there is a difficulty in obtaining reliable experiments with flat-headed projectiles with service guns. Colonel Owen, R.A., made some experiments at Shoeburyness, in 1864, with flat-headed and cylindro-conoidal-headed shot fired from the Armstrong B.L. gun;† and the conclusion drawn from them is

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* It may be noticed, also, that the resistance which drifts the projectile from the direction of motion is greater during the first and second quarters of gyration than during the third and fourth quarters, owing to the greater velocity which the projectile has at the first part of its trajectory.

† *Vide* a paper on the "Derivation of Elongated Projectiles," ("Proceedings, R.A. Institution," Vol. IV. p. 180); also Owen's "Modern Artillery," p. 257.

"that the derivation of cylindrical or flat-headed projectiles* is in the opposite direction to that of projectiles with conoidal or pointed heads, fired with similar rotation. For instance, if a cylindro-conoidal and a flat-headed shot be both fired with right-handed rotation, the derivation of the former will be to the right and the latter to the left."

There seems to be no doubt that flat-headed projectiles, for direct fire under service conditions with right-handed rotation, drift to the *left;* but for indirect and high-angle fire they may be driven to the right. It depends upon the direction of the rotation impressed on the projectile by the resultant of the resistance of the air.

Varying conditions which affect the drift.

All possible conditions may be stated thus (supposing right-handed rotation of the projectile) :—The resistance of the air, according to the shape of the head of the projectile, and the angle between its axis and the tangent to the trajectory, tends

Either to raise the head, (1), or to depress the head, ...(2)
according as C is before or behind G (Fig. 1).
If the 1st, the axis of the projectile commences to gyrate to the right; .....(3)
if the 2nd, the axis commences to gyrate to the left. .....(4)

In the case of (3), if the resultant of the resistance makes a greater angle with the axis of the projectile than the axis of the projectile makes with the tangent to the trajectory (as in Fig. 1), the projectile will drift for the first half-gyration to the right, in the second half-gyration to the left, but the drift on the whole will be to the *right,* for reasons before mentioned; but if the resultant makes a smaller angle, the projectile will drift for the first half-gyration to the *left,* in the second half-gyration to the right, but the drift on the whole will be to the *left.*

In the case of (4), if the resultant makes a greater angle with the axis of the projectile than the axis of the projectile makes with the tangent to the trajectory, the projectile will drift for the first half-gyration to the *left,* in the second half-gyration to the right, but the drift on the whole will be to the *left;* but if the resultant makes a smaller angle, the projectile will drift for the first half-gyration to the *right,* in the second half-gyration to the left, but the drift on the whole will be to the *right.*

It will be seen from the above how very complicated the question of the drift of elongated projectiles becomes, on account of the varying conditions of shape of head, steadiness of the projectile, &c.

Practical conclusions.

The practical conclusions with regard to drift are :—

(1) With service ogival-headed projectiles and right-handed rotation, the drift is to the *right.*

* By "flat-headed projectiles" are meant cylindrical-shaped projectiles |_____|. If the shoulder is rounded off, the centre of resistance may be in front of the centre of gravity |_____), and the drift would be to the right.

(2) With left-handed rotation, under similar conditions, the drift is to the *left*.

(3) With flat-headed projectiles, the drift with right-handed rotation is generally to the *left*.

For the method of determining the drift for any particular piece, and the allowance made to compensate for it in the sighting, *vide* "Treatise on Construction and Manufacture of Ordnance," 1877, pp. 56, 57.

When very low charges are used in high-angle firing, it is found that the projectile strikes the ground base first; with a little more charge, the projectile falls on its side; and with a still higher charge, the projectile strikes the ground nearly point first.

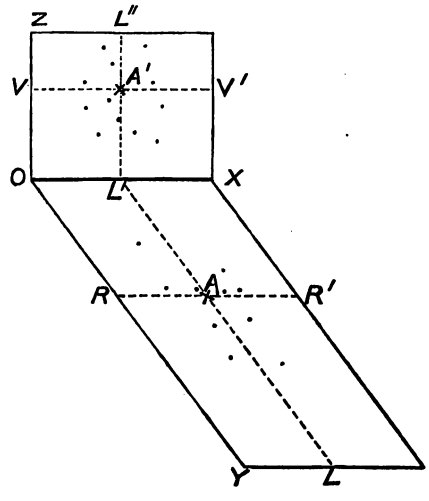
CHAPTER VIII.

Accuracy and Probability of Fire.

Mean Range.—Mean Lateral Deviation.—Mean Vertical Deviation.—Mean Error in Range, or Mean Longitudinal Error.—Mean Lateral Error.—Mean Vertical Error.—Mean Absolute Error.—Method of Estimating the Accuracy of Small-Arms and Ordnance.—Example of Testing Guns for Range and Accuracy.—Zones of Fire.—Probable Rectangles.—Table of Probability Factors.—The Probability of Hitting a Target of given Dimensions.—Causes which Affect Accuracy of Fire.—Variations in Muzzle Velocity.—Unsteadiness of the Projectile as it leaves the Bore and during Flight.—Wind.—Personal Error in Laying.—Inclination of the Trunnions to the Horizon.—Unsteadiness of Gun, Carriage, and Platform on Firing.

In order to judge of the relative accuracy of fire of different guns, it is necessary to fire a large number of rounds from each gun, and compare the results. It is usually done by firing at a horizontal target, and if possible at a vertical target.

The mean range is obtained by dividing the sum of the ranges of **Mean range.**



each shot by the number of rounds fired. Thus, RR' represents the line of mean range.

Mean lateral deviation. The mean lateral deviation is obtained by dividing the algebraical sum of the lateral deviations of each shot by the number of rounds fired. Thus, LL' represents the line of mean lateral deviation.

Mean vertical deviation. The mean vertical deviation is obtained by dividing the sum of the vertical deviations of each shot from the horizontal line OX by the number of rounds fired. Thus, VV' represents the line of mean vertical deviation.

The intersection of the lines RR' and LL' gives A, the point of mean impact on the horizontal target; and the intersection of the lines $L'L''$ and VV' gives A', the point of mean impact on the vertical target.

Mean error in range, or mean longitudinal error. The mean error in range, or the mean longitudinal error, is obtained by taking the numerical sum of the differences between the range of each shot and the mean range, and, dividing by the number of rounds fired.

Mean lateral error. The mean lateral error is obtained by taking the numerical sum of the differences between the lateral deviation of each shot and the mean lateral deviation, and dividing by the number of rounds fired.

Mean vertical error. The mean vertical error is obtained by taking the numerical sum of the differences between the vertical deviation of each shot and the mean vertical deviation, and dividing by the number of rounds fired.

Mean absolute error. The mean absolute error is obtained by taking the sum of the radial distances of each shot from the point of mean impact, and dividing by the number of rounds fired.

The mean absolute error may be estimated both on the horizontal and on the vertical target.

Method of estimating the accuracy of small-arms and ordnance. In order to estimate the relative accuracy of small-arms, the *mean absolute error* is taken on a vertical target. To estimate the relative accuracy of ordnance, the *mean longitudinal, lateral,* and *vertical errors* are observed.

Example of testing guns for range and accuracy. As an example of testing guns for range and accuracy, a direct comparison of the shooting of a long 40-pr. and a short 40-pr. M.L. gun is given in detail. The length of bore of the long 40-pr. was 22 calibres; that of the short 40-pr. was 18 calibres.

Date, and wind.	Rounds.	Gun.	Elevation.	Mean time of flight.	Ranges.			Mean error in range.	Mean lateral deviation.	Mean lateral error.
					Min.	Max.	Mean.			
			°	secs.	yds.	yds.	yds.	yds.	yds.	yds.
30. 1. 74. → 2 to 4	10	Short	3	4·32	1513	1581	1547	17·2	3·08 R.	0·42
"	10*	Long	3	4·44†	1558	1698	1637	30·2	4·20 "	1·38
" → 0 to 1	10	Short	5	6·64	2248	2315	2278	15·8	8·24 "	1·33
"	10	Long	5	6·81	2357	2413	2387	17·9	9·02 "	1·23
5. 2. 74. ← ↖ ↗ ↑ 1 to 2	10	Short	10	12·24‡	3689	3868	3736	41·4	9·22 "	3·06
"	10	Long	10	12·47§	3794	3973	3898	43·6	12·10 "	3·44

* Mean of 9 rounds; 1 omitted as being irregular.
† " 8 " 1 not observed; 1 omitted as being irregular.
‡ " 8 rounds; 2 not observed.
§ " 9 " 1 "

To show how the mean time of flight, mean range, mean error in range, mean lateral deviation, and mean lateral error are obtained, the result of each of 10 rounds fired from the long 40-pr. gun at 5° elevation is given in detail.

No. of rounds.	Elevation.	Charge.	Weight of projectile.	Time of flight.	Range.		Lateral deviation.		Lateral error.	
					1st graze and mean.	Difference of each range from mean.	Left.	Right.	Left.	Right.
	°	lbs.	lbs.	secs.	yds.	yds.	yds.	yds.	yds.	yds.
1	5	7	38½	6·8	2396	9	—	9	—	·02
2	"	"	"	6·8	2386	1	—	8·4	—	·62
3	"	"	"	6·8	2375	12	—	10	—	·98
4	"	"	"	6·8	2357	30	—	7	—	2·02
5	"	"	"	6·9	2410	23	—	7·2	—	1·82
6	"	"	"	6·7	2361	26	—	10·2	—	1·18
7	"	"	"	6·8	2413	26	—	9	—	·02
8	"	"	"	6·8	2403	16	—	7·4	—	1·62
9	"	"	"	6·9	2401	14	—	9	—	·02
10	"	"	"	6·8	2365	22	—	13	—	3·98
			10	68·1	23867	179	—	90·2	—	12·28
				6·81	2387	17·9	—	9·02	—	1·23

The mean time of flight is 6·81 secs.
Mean range ..... 2387 yds.
" error in range ..... 17·9 "
" lateral deviation ... 9·02 " right
" " error ..... 1·23 "

Probability of Hitting.

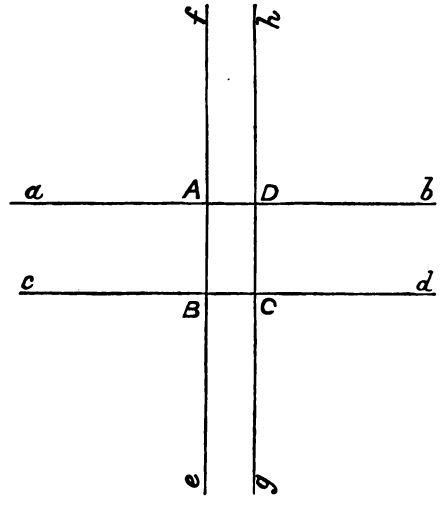
Zones of fire.

By the Theory of Probabilities it may be shown that the dimensions of the zones of fire in which 50 per cent. of the shot would probably strike may be obtained by multiplying the mean errors by the factor 1·69.

If the mean error in range be denoted by R,
" " lateral error " L,
" " vertical error " V,

the depth of the zone containing 50 per cent. of the shot is $1{\cdot}69R$,
" breadth " " " " $1{\cdot}69L$,
" height " " " " $1{\cdot}69V$.

These zones of fire are quite independent of one another, and are bounded by two indefinite parallel straight lines, distant from one another by the amount given above.



Thus, firing at a horizontal target, if R is the mean error in range, then the length included between the two indefinite parallel straight lines *ab* and *cd*, distant $1{\cdot}69R$ from one another, represents the depth of the zone containing 50 per cent. of the shot fired; and if L is the mean lateral error, then the length included between the two indefinite parallel straight lines *ef* and *gh*, distant $1{\cdot}69L$ from one another, represents the breadth of the zone containing 50 per cent. of the shot fired.

The rectangle, *ABCD*, enclosed by the intersection of these zones, represents the area in which 50 per cent. of 50 per cent.—*i.e.*, 25 per cent.—of the shot will probably strike. A vertical rectangle may also be obtained in the same way, if the mean vertical and lateral errors are known. This is the method of calculating the probable rectangle usually adopted on the Continent. Probable rectangles

A comparison of rectangles made out from the results obtained by firing different guns, affords a good criterion of their respective accuracy of fire.

These rectangles are called *probable rectangles*.

The dimensions of the British probable rectangle hitherto adopted are 1·56 times larger than those of the former, but it is proposed in future to adopt the probable rectangle in use on the Continent.

The British probable rectangle represents the area in which 50 per cent. of the shot will probably strike—*i.e.*, twice the probable number of shots contained in the Continental rectangle.

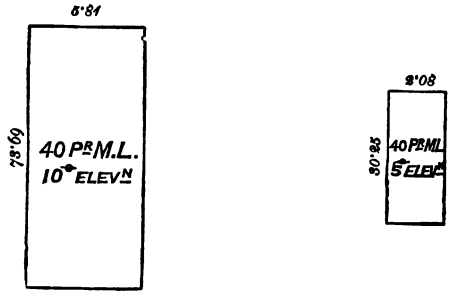
The sides of the British 50 per cent. probable rectangle may be directly obtained by multiplying the mean errors by $1{\cdot}69 \times 1{\cdot}56 = 2{\cdot}64$.

As the Continental method is simpler for calculation, an example of its employment will be given in the case of the 40-pr. M.L. (long) gun at 5° elevation. (*Vide* p. 95.)

The length of the 25 per cent. probable rectangle = 17·9 × 1·69 = 30·25 yds.
" breadth " " " " = 1·23 × 1·69 = 2·08 "

And at 10° elevation :—

The length of the 25 per cent. probable rectangle = 43·6 × 1·69 = 73·69 yds.
" breadth " " " " = 3·44 × 1·69 = 5·81 "



These rectangles represent the area (horizontal) in which 25 per cent. of the shot will probably strike.

When the mean errors of a gun at a given range are known, the probability of hitting a target of given dimensions can be calculated by means of a Table of Probability Factors.

The following table gives factors by which the dimensions of the zones containing 50 per cent. of the shot must be multiplied in order to determine the per-centage of hits on a target of given dimensions :— Table of Probabili Factors.

Table of Probability Factors.

Per cent.	Factor	Per cent.	Factor	Per cent.	Factor	Per cent.	Factor	Per cent.	Factor
1	·02	21	·40	41	·80	61	1·27	81	1·94
2	·04	22	·41	42	·82	62	1·30	82	1·98
3	·06	23	·43	43	·84	63	1·33	83	2·03
4	·07	24	·45	44	·86	64	1·36	84	2·08
5	·09	25	·47	45	·89	65	1·39	85	2·13
6	·11	26	·49	46	·91	66	1·42	86	2·18
7	·13	27	·51	47	·93	67	1·45	87	2·24
8	·15	28	·53	48	·95	68	1·48	88	2·30
9	·17	29	·55	49	·98	69	1·51	89	2·37
10	·18	30	·57	50	1·00	70	1·54	90	2·44
11	·20	31	·59	51	1·02	71	1·57	91	2·52
12	·22	32	·61	52	1·04	72	1·60	92	2·60
13	·24	33	·63	53	1·07	73	1·64	93	2·69
14	·26	34	·65	54	1·09	74	1·67	94	2·78
15	·28	35	·67	55	1·12	75	1·71	95	2·91
16	·30	36	·70	56	1·14	76	1·74	96	3·04
17	·32	37	·72	57	1·17	77	1·78	97	3·22
18	·34	38	·74	58	1·19	78	1·82	98	3·45
19	·36	39	·76	59	1·22	79	1·86	99	3·82
20	·38	40	·78	60	1·25	80	1·90	100	∞

The probability of hitting a target of given dimensions.

Take for example the case of the 40-pr. M.L.R. gun at 5° elevation. The depth of the zone containing 50 per cent. of the shot is 30·25 yds.; the breadth of the zone containing 50 per cent. of the shot is 2·08 yds.

What per-centage of shot from the 40-pr. M.L.R. at 5° elevation at a mean range of 2387 yds. will probably hit a horizontal target 40 yds. long by 4 yds. broad?

The probability factor for length is obtained by dividing the length of the target by the depth of the zone containing 50 per cent. of the shot. Thus,

$$\text{probability factor for length} = \frac{40}{30{\cdot}25} = 1{\cdot}32.$$

By reference to the Table of Probability Factors it will be seen that 1·32 corresponds to 62·7 per cent., which shows that 62·7 per cent. of the shot will probably fall into a zone 40 yds. in depth and of indefinite breadth.

The probability factor for breadth is obtained by dividing the breadth of the target by the breadth of the zone containing 50 per cent. of the shot. Thus,

$$\text{probability factor for breadth} = \frac{4}{2{\cdot}08} = 1{\cdot}92.$$

By reference to the Table of Probability Factors, it will be seen that 1·92 corresponds to 80·5 per cent., which shows that 80·5 per cent. of the shot will probably fall into a zone 4 yds. in breadth and of indefinite length.

The product of the per-centages thus found will give the per-centage of hits on the target. Thus, probable number of hits = 62·7 per cent. of 80·5 per cent. = 50·5 per cent.

The same method applies for determining the per-centage of hits on a vertical target, if the mean vertical and lateral errors of the gun are known.

ERRORS OF FIRE ARE DUE PRINCIPALLY TO

Causes which affect accuracy of fire.

(1) Variations in muzzle velocity.
(2) Unsteadiness of the projectile as it leaves the bore, and during flight.
(3) Wind.
(4) Personal error in laying.
(5) Inclination of the trunnions to the horizon.
(6) Unsteadiness of gun, carriage, and platform on firing.

Variations in muzzle velocity.

It is important, in order to obtain great accuracy of fire, to have the muzzle velocity as uniform as possible. Variations in muzzle velocity result from want of uniformity in the explosion of the charge, which may result from the quality of the powder, the amount of moisture in the powder, or the variation in the cubic space occupied by the charge, and the shape of the cartridge. For instance, the degree of hardness in ramming home will cause a variation in muzzle velocity in those guns where there is not a special provision to stop the projectile at a fixed distance. Another cause of variations in muzzle velocity is the small difference in the weight of projectiles and charges.

The temperature of the gun affects the muzzle velocity. Generally, the first round is short, owing to the muzzle velocity being below the average in a cool gun.

Unsteadiness of the projectile as it leaves the bore and during flight.

The unsteadiness of a projectile as it leaves the bore is the most important source of errors in fire. It may result from accidental causes—such as the shearing of the studs, the breaking up of the gas-check, or from the contact of the body of the projectile with the lands of the bore. In B.L. guns it may result from the lead coat stripping; but in M.L. guns it more commonly arises from non-centring, or insufficient velocity of rotation, or eccentricity of the projectile, and in some cases on the system of rifling employed. This unsteadiness causes greater loss of velocity during flight, since the projectile then presents a greater surface for resistance than when the axis is steady, hence the range is diminished; also, since the drift is allowed for under normal conditions by the inclination of the tangent sights to the left, any unsteadiness of the projectile at the muzzle or during flight will cause a variation in the actual drift, which results in both lateral and vertical error.

Wind.

The variable velocity of wind causes errors in fire which must be allowed for in laying the gun. The deflection leaf on the tangent sight should always be used in making allowance for error due to wind, and the gun should be laid on the object.

Didion gives a formula for calculating the error due to wind;* but as the variability of wind in a long range, both in velocity and direction, is often considerable, no reliable results can be obtained by theory.†

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* *Vide* Didion's "Traité de Balistique," p. 392.

† Colonel Maitland has written a paper on this subject in "Proceedings, R.A. Institution," Vol. VIII. p. 343.

Personal error in laying.

Errors in laying are incident to all systems of laying or sighting, and may be reduced to a small quantity by careful and intelligent gunners when the object can be clearly seen. Constant practice at different ranges is the best way of acquiring this great essential of the gunner's art.

Inclination of the trunnions to the horizon.

Owing to the gun being sighted when the trunnions are horizontal, there arises an error in fire when the trunnions are not horizontal—*i.e.*, when the gun-carriage stands on uneven ground. If the gun is laid by the sights, the projectile deflects towards the side of the lower trunnion.

If h is the height of the perpendicular tangent sight for a given elevation, and the axis of the trunnions is inclined at an angle α, the right trunnion down, the deflection leaf must be increased by $h \sin \alpha$; if, on the contrary, the axis of the trunnions is inclined at an angle α, the left trunnion down, the deflection leaf must be diminished by $h \sin \alpha$.

Unsteadiness of gun, carriage, and platform on firing.

Unsteadiness of the gun and carriage may result from too large a charge being used for the weight of the gun and carriage, causing an excessive jump. The nature of the ground, or platform on which the carriage stands, will have some effect on the amount of jump; also the method of applying the brake on the carriage to check the recoil. If the brake does not act exactly at the same time in each round, there will probably be a variation in the jump of the gun, which will cause an error in fire.

CHAPTER IX.

The Penetration of Projectiles.

Energy absorbed in Penetration.—Energy absorbed in Perforation.—Method of estimating the Resistance to the Projectile.—Formulæ for calculating Perforation of unbacked wrought-iron Plates.—Materials used for Armour-Plates: Cast-iron, Wrought-iron, Steel.—Compound Plates.—The form of Head of Projectile.—Energy required to Perforate a Plate obliquely.—Material of Projectile.—Armour Plates with intervals between them.—The Effect of Common Shell on Armour-Plates.—The Effect of Palliser Shell with wrought-iron points.—Wood.—Earth.—Masonry.—Concrete.

No accurate experiments have as yet been carried out in order to determine the coefficients of resistance for the different materials of projectiles and plates, so that it is difficult to study the subject advantageously in a rigid mathematical way; but practically the amount of penetration, whether for iron or steel plates, or masonry, or earth, has been determined by actual experiment. Various empirical laws have, from time to time, been brought forward, which have sufficed to give approximate results for experimental purposes, but they have not stood the test of any general application. There are, indeed, so many varying conditions which take place in actual experiment that it is almost impossible to apply practically strictly analytical investigations —for instance, the varying qualities of resistance both in projectiles and targets, the variation in shape of the projectile on impact, the possibility of the projectile breaking up, and the amount of heat developed on impact. Such investigations may be studied in Didion's "Traité de Balistique," p. 285 *et seq.*; but in the following pages the subject will be considered in a more experimental way, and the penetrative effect of projectiles against iron, steel, wood, earth, and masonry will be considered in detail. Generally speaking, the penetrative effect depends on the shape and material of the projectile, on its energy, and diameter, and the direction with which it strikes the target.

Energy absorbed in penetration.

It is evident that if a projectile has not sufficient energy to perforate a resisting medium, it will penetrate a certain distance until its energy has been absorbed by the resistance.

Suppose, then, W = weight of projectile in lbs.,
V = velocity on impact in feet per second,
R = resistance in lbs.,
s_1 = number of feet penetrated,

then
$$\frac{WV^2}{2g} = Rs_1; \quad \text{.....(1)}$$

if the resistance be supposed uniform.

Energy absorbed in perforation.

But if the projectile has sufficient energy to perforate, the remaining velocity can be measured after perforation by means of a chronograph, and the energy absorbed in perforating the resisting medium

$$= \frac{W}{2g}(V^2 - v_1^2);$$

where v_1 is the remaining velocity immediately after perforation.

Hence, in this case,

$$\frac{W}{2g}(V^2 - v_1^2) = Rs_1, \quad \text{.....(2)}$$

But if R (the resistance to the projectile) be *variable*, the equation must be written

$$\frac{W}{2g}(V^2 - v_1^2) = \int_0^{s_1} Rds. \quad \text{.....(3)}$$

Differentiating (3), considering v and s as variable,

$$R = -\frac{W}{g} v \frac{dv}{ds} = -\frac{W}{g}\frac{dv}{dt},$$

then
$$\int_{t_1}^{0} Rdt = -\frac{W}{g}\int_{v_1}^{V} dv;$$

or if t_1 = the time of perforation through the resisting medium (resistance supposed *uniform*),

$$R = \frac{W}{g}\left(\frac{V - v_1}{t_1}\right). \quad \text{.....(4)}$$

Substituting in (2), then

$$\frac{W}{2g}(V^2 - v_1^2) = \frac{W}{g}\left(\frac{V - v_1}{t_1}\right)s_1,$$

or
$$t_1 = \frac{s_1}{\frac{1}{2}(V + v_1)}; \quad \text{.....(5)}$$

or the time of passing through the resisting medium

$$= \frac{\text{thickness of resisting medium}}{\text{mean velocity of projectile in passing through the resisting medium}}.$$

If v_{11} is the velocity which would be required *just to perforate* the resisting medium, it follows that

$$\frac{W}{2g} v_{11}^2 = Rs_1;$$

but
$$Rs_1 = \frac{W}{2g}(V^2 - v_1^2) \text{ from (2)},$$

hence
$$v_{11} = \sqrt{V^2 - v_1^2}.$$

The mean resistance, R, is the total force acting on the projectile in lbs.; and knowing R, the resistance in lbs. can be calculated either per inch of circumference or per square inch of sectional area. For instance: suppose $R_{,}$ is the resistance per square inch of projectile's sectional area, and $R_{,,}$ the resistance per inch of projectile's circumference, then

Method of estimating the resistance to the projectile.

$$R = R_{,}\tfrac{1}{4}\pi d^2 = R_{,,}\pi d,$$

$$\text{and } R_{,,} = \tfrac{1}{4}dR_{,};$$

where d is the diameter of the projectile in inches.

So that if the energy per square inch of projectile's sectional area be required, so as just to perforate a resisting medium, we have

$$\frac{Wv_{,,}^2}{2g} \cdot \frac{4}{\pi d^2} = R_{,}s_{,}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

or if the energy per inch of projectile's circumference be required,

$$\frac{Wv_{,,}^2}{2g} \cdot \frac{1}{\pi d} = R_{,,}s_{,}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

The question then arises, which method most nearly agrees with the results of experiment? For thin plates, which can be easily perforated, experiment seems to show that the projectile acts as a punch, cracking the plate *circumferentially*, as in Fig. 2, at the same time slightly bulging the plate, which commences to crack on the inside during perforation, as in Fig. 1. In this case the method of estimating the resistance per inch of projectile's circumference gives the

Fig. 1.

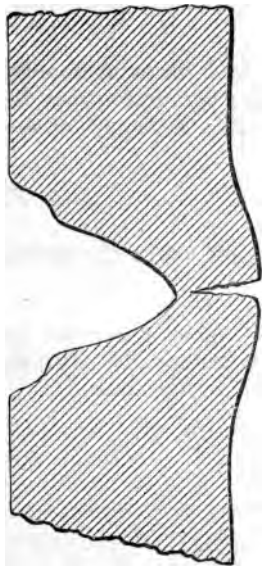
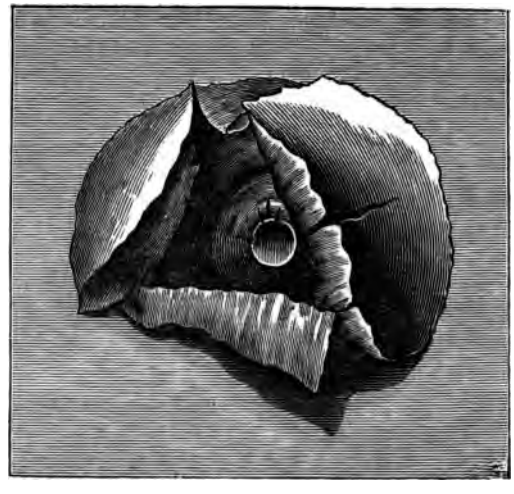


Fig. 2.



best results; but in cases where the projectile acts as a wedge and drives the resisting material before it—*e.g.*, in the case of penetration into earth—the method of estimating the resistance per square inch of sectional area gives approximately the best results. The truth probably lies between these methods of estimation, inclining more to the one or the other according as a punching or a wedging action is brought into play.

Formulæ for calculating perforation of unbacked wrought-iron plates.

The formulæ which have given good results within certain limits on the supposition that the thickness to be perforated may be estimated from the energy per inch of projectile's circumference, are of the form

energy in ft. tons per inch of circum. $(E) = k \times \{\text{thickness of plate in ins. } (s)\}^x$;

where k and x are coefficients which must be determined by experiment.

The formula adopted in the Department of the Director of Artillery, given by Major W. H. Noble, R.A., is

$$E = ks^x,$$

where $k = 2{\cdot}53$ and $x = 1{\cdot}6$; so that

$$s = \left(\frac{E}{2{\cdot}53}\right)^{\frac{1}{1{\cdot}6}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

These constants are computed for ogival-headed Palliser projectiles* and wrought-iron plates, and within certain limits give approximately correct results for plates that can be perforated. Captain Andrew Noble, F.R.S., gives

$$s = \left(\frac{E}{3{\cdot}18}\right)^{\frac{1}{1{\cdot}6}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Colonel Maitland, R.A., has calculated a formula from experimental data, on the supposition that the thickness to be perforated may be estimated by the energy per square inch of projectile's sectional area—viz.,

$$s = \frac{E}{(4{\cdot}733)\pi r^2} = \frac{E}{14{\cdot}87r^2}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where E represents total energy in ft. tons on impact, and r the radius of the projectile in inches.

As an example of calculating the thickness of plate that may be perforated, suppose the energy per inch of circumference of a Palliser shell on impact is known to be 165·6 ft. tons; then by formula (1),

$$s = \sqrt[1{\cdot}6]{\frac{165{\cdot}6}{2{\cdot}53}} = 13{\cdot}6 \text{ ins.}$$

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* More correct results are given with steel projectiles.

It is found that this formula gives good results when the plate can be perforated; but for a very thick plate which cannot be perforated it does not follow that the penetration of the shell will be 13·6 ins., but the result may be approximated to in many cases by allowing a deduction of about 10 per cent.—*i.e.*, the actual *penetration* of the above shell into a wrought-iron armour-plate of considerable thickness may be approximated to at about 12·3 ins.

Table showing Approximate Energy in foot tons per inch of Shot's Circumference required for the Perforation of Unbacked Iron Plates by Steel Shell.

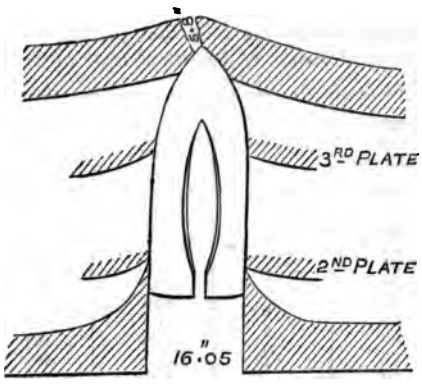
Thickness of plates.	Energy per inch of circumference.	Difference.	Thickness of plates.	Energy per inch of circumference.	Difference.
ins.	ft. tons.		ins.	ft. tons.	
4	25		22·5	368	13
4·5	29	4	23	381	13
5	34	5	23·5	395	14
5·5	39	5	24	409	14
6	45	6	24·5	423	14
6·5	51	6	25	437	14
7	57	6	25·5	451	14
7·5	63	6	26	465	14
8	70	7	26·5	479	14
8·5	77	7	27	494	15
9	84	7	27·5	509	15
9·5	92	8	28	524	15
10	100	8	28·5	539	15
10·5	108	8	29	554	15
11	117	9	29·5	569	15
11·5	126	9	30	585	16
12	135	9	30·5	601	16
12·5	144	9	31	617	16
13	153	9	31·5	633	16
13·5	162	9	32	649	16
14	172	10	32·5	665	16
14·5	182	10	33	682	17
15	193	11	33·5	699	17
15·5	204	11	34	716	17
16	215	11	34·5	733	17
16·5	226	11	35	750	17
17	237	11	35·5	767	17
17·5	248	11	36	785	18
18	259	11	36·5	803	18
18·5	270	11	37	821	18
19	282	12	37·5	839	18
19·5	294	12	38	857	18
20	306	12	38·5	875	18
20·5	318	12	39	893	18
21	330	12	39·5	911	18
21·5	342	12	40	929	
22	355	13			

Calculated from the equation $E = 2{\cdot}53s^{1{\cdot}6}$ (Major Noble's formula), E being the energy in foot tons per inch of the shot's circumference required to perforate a plate of s ins. in thickness.

It has been ascertained by experiment that projectiles of similar forms of head and different calibres require approximately the same amount of energy per inch of circumference to perforate the same armour when the plate is not much thicker than the diameter of the projectile; but when the plate is more than one-fourth thicker than

the diameter of the projectile, the energy per inch of circumference must be in a much higher proportion in order to obtain perforation.

The 80-ton gun was fired against a target consisting of four 8-in. plates, with 5-in. layers of teak between each plate. This target had a total thickness of 32 ins. of iron. The energy per inch of projectile's circumference was about 589 ft. tons, which should according to formula (1) perforate a solid iron plate without backing of 30·16 ins.



The actual penetration was 27 ins., but "the point was visible through large cracks in the back of the target, in some places open to a width of 2½ ins. There was about 5·8 ins. of iron still in advance of the point, but this was fissured and opened in a star crack. The back plate was bent back nearly 14 ins."*

Types of Ships' Armour, with the Energy per Inch of Circumference required for Perforation.

Inches of iron in target.	Inches of wood in target.	Skin of iron.	Energy per inch for perforation.	Inches of iron in target.	Inches of wood in target.	Skin of iron.	Energy per inch for perforation.
		ins.	ft. tons.			ins.	ft. tons.
4·5	10	none	34	12	12	1½	182
5	9	½	42	13	14	1½	204
5·5	10	1	51	14	12	1½	226
6	12	½	60	15·5	12		237
6	30	none	70	14	10	1½	259
8	10	¾	84	17	19	1½	306
8·5	15	none	96	18	18	1½	330
9	10	1	112	21·5	14	1	395
10	9	1¼	126	22	18	1½	437
11	12	1	144	24	12	1½	465
9	0	1½	153				

N.B.—Tables showing the energy of projectiles are given in the "Treatise on the Construction of Ordnance."

* *Vide* the "Engineer," May 11, 1877 ("The 80-ton Gun").

The different materials used in the construction of armour-plates—*e.g.*, cast-iron, wrought-iron, and steel—will next be considered with respect to their comparative advantages or disadvantages for resisting the fire of heavy guns. Materials used for armour-plates.

There are two methods of destroying armour-plates, which may be employed according to circumstances—viz., *punching* and *racking*.

For *punching*, a high velocity is given to the projectile, of a comparatively small diameter, in order to drive it completely through the armour-plate, when the fragments of the projectile take effect on the guns, &c., behind it, and make a hole which if near the water-line of an iron-clad ship soon places it *hors de combat*. If, however, the armour-plate is so thick that it cannot be perforated, *racking* may be resorted to—*i.e.*, expending the energy of the projectile on cracking the armour and shaking it from its supports. This may be done most effectually with projectiles of comparatively large diameter and low velocities; the object being in this case not to penetrate but to break up the armour. It will be seen (p. 109) that wrought-iron plates yield to punching and resist racking, while steel plates of similar thickness yield to racking and resist punching.

The effect of projectiles on a cast-iron target is very destructive: the cast-iron cracks and flies, and in its ordinary condition it is not suitable for employment for defensive purposes. Chilled iron armour may be used in some places, where it is not likely to be subjected to continued bombardment, but the thickness required to resist fracture is much greater than that required by wrought-iron under similar circumstances. Cast-iron.

The effect produced by a projectile on a wrought-iron target is less destructive than upon cast-iron. Owing to the superior toughness and malleability of wrought-iron, there is not such a tendency to crack, and break up into fragments. A large number of experiments have, from time to time, been carried out at Shoeburyness in order to test the requisite thickness of wrought-iron plates to resist perforation.* Wrought-iron.

Iron plates are made either with the requisite thickness of iron in a single solid plate, or the thickness is made up of two or three iron plates interlined with wood. The former are called *solid plates*, the latter *laminated plates*.

There are differences of opinion as to the relative merits of the two systems, when applied to the protection of ships or batteries. On the one hand, it appears that the laminated plate is more easily perforated than a solid plate of the same thickness. Colonel Inglis, R.E., considers that the resistance of armour-shields 7 ins. thick, composed of one, two, or three equal layers, are respectively as 100, 96, and 89; and that a single plate 17½ ins. thick is about equal to three 6½-in. plates separated by 5-in. layers of teak. On the other hand, the laminated plate system admits the employment of a wider plate; the plates themselves being more easily and soundly manufactured. In

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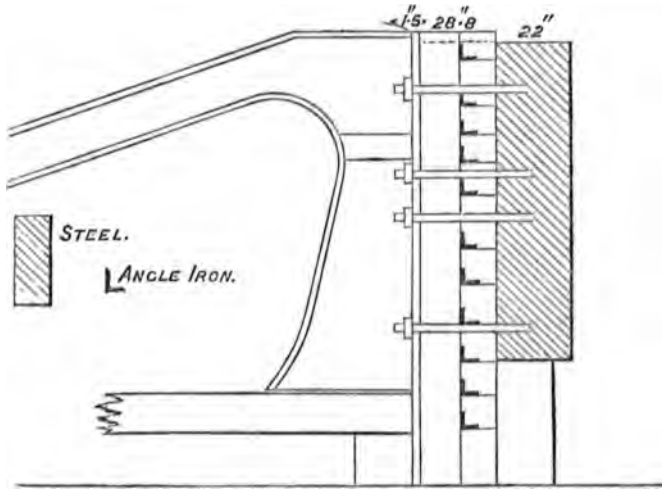
* *Vide* "Extracts from the Proceedings of the Department of the Director of Artillery."

turret ships this is important, as armour-plates can be made of a breadth equal to the height of the turret, and through joints can thus be dispensed with, the bolting and securing of the structure is facilitated, and the absolute cost is less.

Steel. Armour-plates made of steel have some very important advantages which point to the advisability of employing steel instead of wrought-iron, either partially or entirely, for defensive purposes. Steel plates have a greater absolute tenacity, and more perfect homogeneity of structure than wrought-iron plates. They have also a greater resistance to penetration. This latter is of great importance, as it becomes possible by the use of steel armour to prevent ships' sides being punched or *perforated* by means of the heaviest gun yet made. There is, however, a great disadvantage which renders steel armour more susceptible of succumbing under the system of demolition by racking —*i.e.*, a crystalline structure which makes it liable to crack under relatively light blows; so that by repeated blows from medium guns the demolition of the steel plate is certain.

Experiments carried out by the Italian Government at Spezia in the autumn of 1876 with the 100-ton gun proved conclusively the value of the Schneider steel plates over wrought-iron plates of the same thickness. The Schneider steel target, which successfully resisted the 100-ton gun, striking with an energy of about 589 ft. tons per inch of circumference, consisted of a solid plate of soft hammered steel 22 ins. thick (*vide* figure), supported by two layers of wood backing—one vertical, the other horizontal. The wood backing was again supported by an iron plate 1·5 in. thick, and behind this again by the vertical frames of the ship, supported by the proper deck beams, those above being iron, and bent down to meet the ground at an acute angle; thus gaining the support which would be given by the other side of the



ship.* The bolts which passed through the backing to hold on the armour did not perforate the plate entirely; they were only screwed in about as far as the centre of the thickness of the steel. The first or horizontal layer of wood backing was further strengthened by angle-iron.

Another target of similar dimensions, so far as thickness of plate and backing are concerned, was made, having a front plate of 22 ins. of wrought-iron instead of steel, and the whole structure arranged in a similar manner to the steel target.

These two targets were subjected to the fire of 10-in. and 11-in. guns. The effect of these comparatively light guns on the Schneider steel target was to crack it. These cracks gradually extended, and the destructive effect of repeated shocks from these guns proved conclusively that the steel armour was quite unsuited for resistance to continued firing from these guns.

The wrought-iron target resisted the fire more successfully. Slight cracks occurred in the immediate proximity of the part struck; but these cracks did not spread, and there were no signs of cracks in other parts. The extreme penetration was 18 ins.; so that the wrought-iron target was quite strong enough to withstand the fire of 10 and 11-in. guns. Yet this target was completely perforated by the 100-ton gun, while the steel target was more successful in resisting the blow; so that the Committee of Italian officers at Spezia unanimously considered that a solid plate of Schneider steel is more advantageous than wrought-iron of similar thickness as armour-plating for ships' sides.†

Compound plates.

Experiments have since been made at Portsmouth with the view of combining the greater resistance to perforation offered by steel with the greater tenacity of wrought-iron. The advantage of wrought-iron as a material for armour-plates is that the effect of the blow is localised, and consequently answers better under the fire of ordinary heavy guns than steel. Plates thus made up of different metals are called *compound plates*. The object sought after is to have a hard steel surface welded on to soft wrought-iron, to prevent the steel from splitting.‡

A Wilson compound plate, consisting of a 5-in. hard-steel plate welded on to a 4-in. wrought-iron plate, was tested at Shoeburyness. The steel front was struck by a 7-in. Palliser shell, which broke up on impact, and penetrated to a depth of 3·2 ins., cracking the steel armour, but it did not extend to the iron. The penetration is much less than would have occurred in an ordinary wrought-iron plate of good quality; showing that on account of the breaking up of the projectile on the hard-steel plate, there was a considerable loss of work on the plate. The wrought-iron plate prevented the steel plate cracking so much as it would have done if the plate had wholly consisted of steel.

Other experiments have not been so satisfactory for the compound armour-plate; but it is probable that some combination of wrought-iron and steel will be employed as armour for ships' sides.

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* *Vide* letter from Special Correspondent of the "Times," dated Spezia, October 25, 1876.

† *Vide* p. 474, "Mittheilungen über Artillerie und Genie Wesen," 1877.

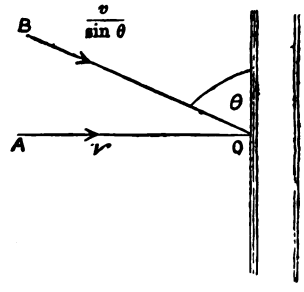
‡ *Vide* a paper in the "Proceedings of the Institution of Mechanical Engineers," "On the Construction of Armour to resist Shot and Shell," by Captain C. O. Browne, late R.A.

The form of head of projectile.

In order to investigate more fully the question of the penetration of armour-plates, it is necessary to consider the form of head and the material of the projectiles employed.

Several forms of head have been experimented with in this country —viz., flat, ogival, hemispherical, and conoidal. The ogival head struck with a radius of $1\frac{1}{2}$ diameters has been adopted for Palliser projectiles, as experiment has shown that this is the most suitable form for the penetration of armour-plates. An ogival head of two diameters has also given good results. The ogival head acts more as a wedge, and it consequently gives better results in penetration than the flat head, especially with thick solid plates. The flat head acts like a punch, driving the material of the armour-plate in front of it. This no doubt increases the resistance which a flat-headed projectile meets with in penetration.

The flat head is superior to the ogival, (1) when firing through water, (2) when firing at extreme angles of oblique fire. In the first case, a ship's side is more likely to be hit below the water-line by a flat-headed than by an ogival-headed projectile, since the ogival-headed projectile has a greater tendency to be deflected upwards by the water; in the second case, the ogival-headed projectile glances off the target where a flat-headed projectile would penetrate. A steel flat-headed projectile will penetrate at an angle of impact with the surface of the plate of about 30°, while the ogival-headed projectile of the best form will hardly penetrate at a less angle than about 40°.



Energy required to perforate a plate obliquely.

If v is the velocity required for a projectile to pierce an armour-plate with right-angled impact AO, its energy per inch of circumference

$$= \frac{Wv^2}{2g \,.\, \pi d}.$$

But if the projectile strikes obliquely, as BO, at an angle of impact θ, then, if it turns in on its point and perforates *normally*, it is readily seen that the velocity for perforation must be $\frac{v}{\sin\theta}$, and consequently the energy per inch of circumference for oblique perforation will be

$$\frac{Wv^2}{2g\pi d \,.\, \sin^2\theta};$$

so that in this case the number of foot tons per inch of circumference to perforate an armour-plate obliquely is found by dividing the

number of foot tons required for perforation by right-angled impact by the square of the sine of the angle of impact.

But if the projectile goes straight through the plate without turning in, the energy per inch of circumference for oblique perforation will be

$$\frac{Wv^2}{2g\pi d \sin \theta}.$$

This formula gives the best results for flat-headed projectiles and comparatively thin armour-plates.

Material of projectile.

Chilled iron, wrought-iron, and steel have been experimented on to ascertain their respective merits for materials to make projectiles for piercing armour-plates. Chilled iron has been adopted in this country, on account of its easier manipulation, its great hardness, and its comparative cheapness of production. The best material of projectile would be that which would neither break up on impact nor change its shape. Any change of shape means a loss of energy uselessly expended in heating the projectile; while if the projectile breaks up, it generally fails to impress all its energy on the target. Steel answers more to these requirements than either chilled or wrought-iron.

Experiments carried out at Shoeburyness with 9-in. projectiles with ogival heads and equal weights, made of steel or chilled iron by different manufacturers, and fired against an armour-plate of wrought-iron 12 ins. thick, have shown that steel is the best material for armour-piercing projectiles. The chilled iron shells break up on impact generally, while the steel shells rarely do so; and if they break up it is always in large pieces, and in some cases the head only separates from the body. The shells made of Whitworth steel give the best results; it was the only sample of steel which fairly pierced the plate and remained perfect without any sensible alteration in shape. The penetration with steel projectiles which neither break up nor sensibly alter their shape is greater than with ordinary chilled projectiles.

Armour-plates with intervals between them.

It has been found, when firing Palliser shell against armour-plates, that the destructive effect may be greatly modified by placing a thin armour-plate unbacked at a short distance in front of the armour-plate proper. The front armour-plate is of course easily penetrated, but in perforation the chilled shell is broken up, and the fragments have but little effect on the armour-plate in rear.

At Shoeburyness, on the 27th March, 1877, a Palliser shell was fired from the 38-ton gun at a 4-in. iron plate placed in front of another 10-in. plate at an interval of 4 ft. 7 ins. The shell passed through the 4-in. plate, making a hole 12·5 ins. in diameter, and breaking up in doing so. A quantity of it was found in a state of "splash" upon the face of the 10-in. plate. The 10-in. plate was comparatively little injured.

However, when firing steel projectiles which do not break up in perforating the thin armour-plate, this no longer holds good. A steel shell fired from a 9-in. gun completely pierced an 8-in. plate placed 4 ft. 6 ins. in rear of a 4-in. plate placed at an angle of incidence of 60°. It thus appears that the method of arrangement of armour-plates with intervals between them is useless for structures liable to

attack by steel projectiles; at the same time, it is a useful fact to remember in case of attack with chilled projectiles.*

The effect of common shell on armour-plates.

The penetration of common shell is considerable, although not to be compared with that of Palliser shell under similar conditions. The large bursting charge in the common shell renders it particularly destructive in cases where the armour-plate can be pierced. It may be fired with or without a percussion fuze. With the general service percussion fuze the burst takes place quicker than when no fuze is used.

The 10-in. M.L. gun, with battering charge and common shell, is capable of perforating a 5-in. iron unbacked armour-plate at 70 yds.

The effect of Palliser shells with wrought-iron points.

It has been ascertained by experiment that if a thin wrought-iron plate is placed in front of a steel plate, the whole structure may be perforated; while if the steel plate is placed in front of the wrought-iron plate, the projectile is broken up on impact and fails to perforate the target. In the latter case, the hard steel breaks up the projectile, and a part of the energy of the projectile is expended in breaking up before it can be impressed on the target; while in the former case, although the projectile may be broken up eventually, yet time is allowed, through the support afforded by the wrought-iron, for the projectile to impress nearly all its energy on the target.

On the same principle it was proposed by Captain English, R.E., to try the effect of a wrought-iron cap placed on the point of a Palliser shell, when firing against steel armour-plates. The wrought-iron cap was wrapped round the head of the shell, and secured by two steel pins fitting into the extractor-holes. The shell so capped was found to penetrate the steel armour, where the uncapped shell, under similar conditions, had broken up into small pieces and failed to penetrate.

Wood.

The effect of a projectile fired against wood depends on the nature of the wood, the direction of the blow with regard to the fibre of the wood, and the diameter of the projectile. Oak is the best wood to resist artillery fire, but its efficiency depends on the way in which the fibre of the wood is disposed. If the fibre is arranged perpendicularly to the general direction of fire (which is the best), there is a tendency in the wood to close up the hole completely when the projectile is of small calibre. With projectiles fired from siege and garrison guns the hole does not close up entirely, and large splinters are torn away from the back of the target.

If the wood is arranged so that the projectile strikes it along the fibre, it is seen to split and splinter even under the fire of field guns.

At about 900 yds. the 16-pr. gun may be expected to perforate 3 ft. 3 ins. of wood, and the 9-pr. gun at the same distance may be expected to perforate 1 ft. 10 ins. of wood.

Earth.

The effect of a projectile on earth can be more easily repaired than in the case of any other material. In general, the earth closes up the hole made; and if the earthwork is thick enough to resist complete penetration, the projectile buries itself in the earth with its point more or less turned round towards the direction from which it was fired.

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* For information on this subject, and on armour-plated targets generally, *vide* "Targets for the Trial of recent Heavy Ordnance," by Colonel T. Inglis, R.E., in the "Occasional Papers of the Royal Engineer Institute."

Penetration into Earth Targets (Sand and Clay).

Date and place of experiment.	Nature of target.	Ordnance.	Projectile.		Charge.	Range.	Striking velocity.	Angle of elevation.	Angle of descent.	Penetration.
			Nature.	Weight.						
				lbs.	lbs.	yds.	f.s.	° ′	° ′	ft. ins.
Tegel, 1870.....	parapet of loam	short 15^{cm} gun	common shell	61·1	3·3	1026	762	4 11	5 15	12 2
" 1872.....	"	15^{cm} steel gun	"	60·3	4·4	656	889	2 0	2 11	16 3
" 1870.....	parapet of sand	short 15^{cm} gun	"	61·1	3·3	1029	752	4 11	5 15	6 2·8
" 1872.....	" traverse of sandy soil, thrown up 6 months before	15^{cm} hooped gun	live common shell	61·0	13·67	2734	935	4 45	6 45	16 3
" 1872...	parapet of sand	12^{cm} bronze gun	common shell	32·2	2·3	1149	810	3 45	4 19	8 5
" 1876.....		heavy field gun	"	15·4	3·3	437	1234	0 23	0 45	3 3·4
" 1876.....	"	light field gun	"	11·2	2·75	"	1286	0 19	0 37	3 3·4
Newhaven, 1863	clay	12-pr. B.L.R.	"	10·4	1·5	1060	987	—	—	4 0
"	"	40-pr. B.L.R.	shot	41·1	5·0	"	1014	—	—	14 9
"	"	7-in. B.L.	"	111·0	12·0	"	972	—	—	21 3
Tegel, 1863	sandy soil	21^{cm} mortar	common shell	176·0	3·3	1564	400	30 0	31 45	2 2
"	"	"	"	"	"	1488	402	60 0	62 7	2 4
"	"	"	"	"	7·7	3883	606	30 0	33 26	1 8
"	"	"	"	"	"	3609	623	60 0	64 15	4 5

N.B.—The German experiments are taken from the "Handbook for Prussian Artillery Officers," translated by Major C. H. Fairfax Ellis, R.A.

The effect of a projectile on masonry is to make a conical hole on the surface, and then a clean cut as far as it will penetrate. The masonry in front is shattered, and pieces fly back some considerable distance towards the breaching battery. The depth of penetration varies with the shape and material of the projectile. Masonry.

Penetration into Masonry (Brickwork).

Date and place of experiment	Nature of target.	Ordnance.	Projectile.			Charge.	Range.	Elevation.	Angle of incidence.	Angle of descent.	Striking velocity.	Energy on impact.	Size of crater.			Amount of masonry displaced.	Masonry displaced		Remarks.
			Weight.	Live or plugged.	Fuze.								Depth.	Breadth.	Height.		per ft. ton of energy.	per lb. of projectile.	
			lbs.			lbs.	yds.	° ′	° ′	° ′	f.s.	f.t.	ins.	ins.	ins.	cub.in.	ins.	ins.	
Graudenz, 1873.	strong brickwork	short 15cm gun	61·2	Live com. shl.	service	3·3	1488	5 37	90 0	7 34	732	265	17·3	55·1	39·6	10983	41	183	
"	very hard brickwork	"	"	"	delayed action	3·3	96	—	74 0 to 82 0	4 3 to 5 0	822	291	25·2	47·5	43·5	14975	51	249	
"	"	"	"	"	service	3·3	96	—	"	"	822	291	27·5	39·6	43·6	13636	47	227	
Stettin, 1868.	good brickwork	15cm gun	"	plugged com. shl.	—	4·4	110	—	66 15	14 0	980	414	39·6	37·1	18·5	8291	20	138	
"	"	"	"	"	—	5·0	110	—	68 30	14 0	1038	464	39·0	37·1	33·9	12925	28	215	
"	"	"	60·3	filled com. shl.	service	4·4	110	—	74 31	14 0	939	366	27·6	47·4	39·5	14598	40	243	
Graudenz, 1873.	very hard brickwork	15cm hooped gun	61·1	plugged com. shl.	—	8·6	186	—	64 0 to 70 0	3 0 to 4 0	1148	568	35·8	33·5	33·6	12988	23	216	Shell remained in hole.
"	"	"	"	"	—	10·8	"	—	"	"	1312	725	39·5	32·3	39·6	13948	19	230	"
"	"	"	"	"	—	13·1	"	—	"	"	1476	938	39·6	47·2	43·5	22940	24	382	Shell broke up on impact.
"	"	"	"	"	—	13·7	"	—	"	"	1525	1000	37·4	43·3	39·4	16736	17	278	"
"	"	"	"	live shell	delayed action	8·6	"	—	"	"	1148	568	39·6	59·1	39·4	25961	46	432	
"	"	"	"	"	service	8·6	"	—	"	"	1148	568	39·6	43·4	51·2	24528	45	409	
"	"	"	"	"	none	8·6	"	—	"	"	1148	568	37·0	67·1	63·0	42137	74	702	Shell exploded in the wall.
"	"	"	"	"	delayed action	10·8	"	—	"	"	1312	725	39·5	55·2	47·4	28575	39	476	
"	"	"	"	"	service	10·8	"	—	"	"	1312	725	39·5	55·1	51·3	30139	41	502	
"	"	"	"	"	delayed action	13·1	"	—	"	"	1476	938	36·6	59·0	59·2	33427	35	557	
"	"	"	"	"	service	13·1	"	—	"	"	1476	938	33·9	74·8	59·3	40887	43	681	
"	"	"	"	"	delayed action	13·7	"	—	"	"	1525	1000	43·3	71·1	63·2	54303	54	905	
"	"	"	"	"	service	13·7	"	—	"	"	1525	1000	39·6	75·0	59·1	50391	50	839	
"	"	"	"	"	none	13·7	"	—	"	"	1525	1000	26·8	55·3	55·1	31790	32	529	Shell exploded in the wall.

Vide "Handbook for Prussian Artillery Officers," translated by Major Ellis, R.A.

With concrete the surface is more broken up than with masonry. Concrete. Concrete generally breaks up in layers, but it is superior to masonry in resisting penetration.

The following table gives the penetration into brickwork and concrete, from experiments carried out at Eastbourne in 1876:—

Material of target.	Nature of ordnance.	Projectile.		Charge.	Range.	Penetration.
		Nature.	Weight.			
			lbs.	lbs.	yds.	
Brickwork	64-pr. M.L.R.	common shell	64	10	1000	3 ft. 2 ins. to 5 ft.
"	"	battering shell	89	12	"	Perforated 7 ft. of wall.
Concrete	"	"	"	10	100	4 ft. 8 ins.
"	"	common shell	64	"	"	2 ft. 6 ins.
"	"	"	"	12	1000	4 ft.
Granite and Portland cement concrete	12·5-in. M.L.R.	Palliser shell	822	130	70	Perforated 11 ft. and destroyed the wall.

APPENDIX I.

INSTRUMENTS FOR DETERMINING THE VELOCITIES OF PROJECTILES, THE RESISTANCE OF THE AIR, AND THE PRESSURE IN THE GUN.

Instruments for Determining the Velocities of Projectiles outside the Gun.—The Principle of Determining the Velocity of a Projectile.—Method of Obtaining Muzzle Velocities.—The Le Boulengé Chronograph.—Bashforth's Gravity Chronograph.—Watkin's Electric Chronograph.—Bashforth's Clock Chronograph, for Determining the Resistance of the Air.—Instruments for Determining the Pressure in the Gun and the Velocity of the Projectile inside the Gun.—The Crusher Gauge.—The Noble Chronoscope.

Instruments for determining the velocities of projectiles outside the gun.

The instruments now in use for determining the velocities of projectiles outside the gun are the chronographs called after their inventors' names, viz.—

Le Boulengé's Chronograph,
Bashforth's Gravity Chronograph,
Watkin's Electric Chronograph.

The principle of determining the velocity of a projectile.

The principle of determining the velocity of a projectile outside the gun is the same in all cases, but the method of registering the results varies in all these instruments. The principle consists in this—viz., setting up two or more screens at known distances apart in connection with an electric battery, and determining by means of electricity the *exact* time of the projectile passing through the respective screens. The interval of time being thus found in which the projectile has traversed a known distance, the mean velocity of the projectile is found by dividing the known distance by the interval of time found by the chronograph. Thus, suppose the extreme distance between the screens is known to be 200 ft., and it has been determined that the projectile took $\frac{1}{6}^{th}$ of a second to traverse this distance; it is plain that the mean velocity of the projectile while traversing the distance of 200 ft. was $6 \times 200 = 1200$ f.s.

This mean velocity thus found is assumed to be the actual velocity of the projectile at half the distance from the first screen—*i.e.*, in the above example at 100 ft. from the first screen. Fortunately, at those velocities where the cubic law holds this assumption is correct—*i.e.*, that the mean velocity observed over the known distance is the actual velocity at half that distance.

Method of obtaining muzzle velocities.

But in order to determine the *muzzle* velocity of the projectile this distance must be added to the distance of the first screen from the

muzzle of the gun—suppose, 50 ft.; then the chronograph registers the velocity of the projectile at a distance of 100 + 50 ft. = 150 ft. from the muzzle of the gun.

Allowance must be made for the loss of velocity of the projectile in traversing the distance between the muzzle of the gun and the place where the velocity has been observed. This can be most simply done by means of the Distance and Velocity Table (p. 55).

The Le Boulengé Crono-graph.

* The Le Boulengé Chronograph consists of a hollow brass column, *S*, which supports two electro-magnets, *A* and *B*, and a small bracket, *K*. The column stands on a triangular base, upon which is fixed the "*trigger*," *T*. (Plate III., Fig. 2.)

The *electro-magnet*, *A*, supports a long cylindrical rod, *C* (Plate III., Fig. 1), suspended vertically, and called the "*chronometer*." This rod is partially covered with two zinc tubes, *D E*, called "*registers*."

The *electro-magnet*, *B*, sustains a shorter rod, *F* (Plate III., Fig. 1), named the "*registrar*."

The "*trigger*" (Plate III., Fig. 2) consists of a circular steel knife, *G*, fixed in a recess of the spring, *H*, by means of the screw, *N*, which forms an axle upon which it can be turned so as to bring a fresh portion of the edge opposite the chronometer.

The spring, *H*, can be "cocked," or restrained, by means of the catch on one end of the lever, *I*.

The other end of this lever carries a disc, *O*, fixed to a screw, by means of which it can be raised or lowered as required.

This disc is vertically below the registrar when suspended to its electro-magnet; consequently, when the current through the second screen is broken, the registrar falls on the disc and releases the spring, *H*.

The tube, *L* (Plate III., Fig. 1), retains the registrar after its fall.

If it be required to alter the time taken by the registrar to release the knife, it is done by raising or lowering the disc of the trigger, by turning it in the direction with the *sun* to *increase* the time, and *against the sun to reduce* it.

The screw has a pitch of one millemètre, and the circumference of the disc is divided by notches into ten equal parts, in which the paul, *P*, works. By this arrangement the disc can be moved any required number of tenths of a millimètre (within certain limits), and is retained in the required position by the paul.

The screw, *M*, passes through the lever, and acts against the fulcrum supporting it. It is intended for regulating the hold of the catch of the lever on the spring, which should always be as light as possible.

This is regulated once for all; but should the spring at any time show a tendency to escape of itself, this defect can be remedied by slightly withdrawing the screw, *M*.

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* Extracted, with slight alterations, from "Proceedings, R.A. Institution," Vol. IX. p. 354. "Instruments employed by the Committee on Explosives for Investigating the Action of Gunpowder in the Bores of Guns, by Captain C. Jones, R.A., Assistant in the Experimental Branch of the Director of Artillery's Department."

The *disjunctor* (Plate III., Fig. 3) is composed of a mainspring, *t*, carrying a cross-piece, *u*, covered with insulating material, and passing under the two steel plates, $q\,q'$. By pressing the milled-headed screw, *z*, the spring is compressed and held by the catch, *x*, allowing the plates, $q\,q'$, to come into contact with the metal pins, $r\,r'$, and thus complete the circuits by bringing the screws, $s\,v$ and $s'\,v'$, into connection with one another. When the catch, *x*, is pressed, the mainspring being released, its cross-piece strikes the two plates exactly at the same instant, raises them from the screws, and thus breaks both currents identically at the same time.

The arrangement of the screens and electric batteries is shown in Plate IV.

There are two screens, 120 ft. apart, through which the projectile whose velocity is required is fired. The first screen is nearest to the gun and at a convenient distance from it, so that the blast of the discharge of the gun may not injure the screen.

The screens are composed of a wooden frame of convenient size, in which are stretched fine copper wires, placed so close together as to ensure the projectile cutting one of the wires. The current from the battery connected with the screen passes through these copper wires. When a wire is cut by the passage of the shot, the current is permanently broken, and the electro-magnet in connection with the screen becomes demagnetised.

There are two batteries, which are totally disconnected from one another. One is called the Chronometer Battery, and is in circuit with the Disjunctor, the first screen, and the chronometer electro-magnet in the chronograph; the other is called the Registrar Battery, and is in circuit with the Disjunctor, the second screen, and the registrar electro-magnet in the chronograph.

When the chronograph is ready for an experiment, the current passes through the respective batteries, screens, and electro-magnets. The electro-magnet *A* is just strong enough to suspend the chronometer, and the electro-magnet *B* the registrar. When the first screen is cut by the projectile the chronometer falls freely in a vertical direction, and when the second screen is cut the registrar falls, and striking the disc on the free end of the lever of the trigger, liberates the spring, which carries forward the knife until it strikes the chronometer in its fall and makes an indent in the upper zinc tube.

A very simple relation exists between the height of this indent and the velocity of the projectile. It is evident that the time which elapses after the fall of the chronometer before the registrar is released is the time taken by the projectile in passing over the distance between the screens; the less, therefore, the velocity of the projectile, the further in advance will the chronometer be, and the higher will be the indent.

A *graduated rule* is used for measuring the height of the indent above the zero point. It is of brass, and is graduated on both edges; the upper edge is a scale of equidistant parts, divided into inches and tenths, reading to thousandths with a vernier, and is intended for use in connection with the tables.

The lower scale is for reading off the velocity of the projectile without any calculation. It is graduated in feet for a distance between the screens of 120 ft.

The zero point on the scale corresponds with the "*origin*," or the point at which the knife marks the zinc if the trigger is set in action when the chronometer is at rest.

The rule is fitted at the zero end with a jointed piece having a slightly conical projection, which enters into a recess in the bob of the chronometer when applied for measuring the marks. Care must be taken not to injure this portion of the scale, or the measurement may be rendered inaccurate.

As stated above, if the trigger be set in action when the chronometer is at rest, a mark will be made by the knife on the zinc, which point we will call the "*origin*," as it is the zero point from which the height of fall of the chronometer must be calculated.

Let H be the height above the origin of the mark obtained by firing a projectile through the screens. Since the chronometer follows the law of the fall of heavy bodies,

$$T' = \sqrt{\frac{2H}{g}},$$

will be the time it was in motion before receiving the impression. Now, T' would be the time required by the projectile to traverse the distance between the screens, supposing that the chronometer commences to fall the instant the projectile passes through the first screen, and further supposing that it is struck by the knife at the precise instant the shot cuts the second screen. But this is not the case. In fact, after the rupture of the first screen, a certain time, θ, elapses before the electro-magnet is demagnetised sufficiently to free the chronometer; the movement of the chronometer will therefore be delayed, and the observed time consequently diminished, by the quantity θ.

Again, some time elapses between the cutting of the second screen and the moment when the knife reaches the chronometer—viz., the time required for the following operations:—

- θ' for the demagnetisation of the electro-magnet supporting the registrar.
- t' for the fall of the registrar to the disc of the trigger.
- t'' for the disengagement of the catch.
- t''' for the knife to pass over the horizontal distance which separates it from the chronometer.

Now, it is evident that the chronometer, before it is struck by the knife, will have been in motion during the sum of the above time in addition to the time taken by the shot in passing over the distance between the screens. Consequently, the observed time T' is too great by the sum of $(\theta' + t' + t'' + t''')$. We have also shown above that T' is too small by the quantity θ—the time required to demagnetise the chronometer electro-magnet. Therefore, to ascertain the true time T,

we must deduct from T' the quantity $(\theta' + t' + t'' + t''' - \theta)$, which we will call t.

We have then $T = T' - t$.

Now, suppose $T = 0$, or, in other words, suppose the shot to cut both screens simultaneously, then we should have $T' = t$. From which it appears that t would be the time recorded on the chronometer if both currents were cut identically at the same instant. This we can do by using the disjunctor; and we thus obtain a mark, on the lower zinc tube, at a height above the origin equal to the space passed over in the time t, which we call the *disjunctor reading*. The time corresponding to this reading must be deducted from the whole time recorded on the chronometer, to arrive at the time taken by the shot to traverse the distance between the screens. As before stated, the disc of the trigger can be raised or lowered so that the disjunctor reading can be altered (if required) within certain limits, and we can thus regulate the instrument so that the time t shall have a constant value. The value of t for which the velocity scale has been calculated is 0·15 of a second, and the height of the corresponding mark above the origin is 4·345 ins. (110·370 mill.) Starting with this assumption, a scale has been calculated for a distance between the screens of 120 ft., by means of which the velocity of the projectile can be at once determined without the aid of any calculation. Should it be necessary to place the screens nearer to one another, the velocity can be found by multiplying the number read off on the scale by the fraction $\frac{D}{120}$, D being the actual distance between the screens in feet.

The method of calculating this scale is as follows:—

Suppose the shot to have a velocity of 1200 ft. a second, it would take $\frac{120}{1200} = 0''{\cdot}1$ to traverse the distance between the screens.

The instrument will therefore mark $0''{\cdot}15$ (disjunctor reading) $+ 0''{\cdot}1$, or $0''{\cdot}25$, and the corresponding height of fall from the origin will be

$$H = \tfrac{1}{2} g T'^2 = \frac{g \times \overline{0{\cdot}25}^2}{2} = 12{\cdot}075 \text{ ins.}$$

Conversely, if the mark on the chronometer is 12·075 ins. above the origin, we know that the velocity of the projectile is 1200 f.s.; the disjunctor reading being at a height corresponding to $0''{\cdot}15$, and the screens 120 ft. apart.

This calculation has been made for a series of velocities increasing from foot to foot for velocities from 850 to 1500 ft. a second, and increasing by 5 ft. from 1500 to 1800 ft. a second, and the corresponding heights engraved on the scale supplied with the instrument.

Bashforth's Gravity Chronograph.

The Gravity Chronograph is a modification of Bashforth's Clock Chronograph (described at p. xii.), which he so successfully used in his experiments on the resistance of the air to projectiles in motion. The Gravity Chronograph is worked by means of a voltaic battery and electro-magnetism. There is only one battery and one electro-magnet, which registers the time of the projectile cutting the screens and gives a scale of time. There is no disjunctor, as the effects of

remaining magnetism are obviated by the very short interval of time that the electro-magnet is magnetised during the experiment. There are three screens, the centre screen serving as a check on the accuracy of the experiment.

The screens are arranged on a different principle to the Le Boulengé screens. In the latter, when the wire of the screen is cut the current is permanently broken, but in the Bashforth screens arrangements are made for a renewal of the current, so that the current may be restored to the electro-magnet in the chronograph before the shot reaches the next screen.

Figs. 1 and 2 give the details of the screens. Fig. 1 represents a piece of board rather longer than the width of the screen to be formed. Transverse grooves are cut at equal distances, something less than the diameter of the shot, as shown in the diagram. Staples of hard brass

Fig. 1.

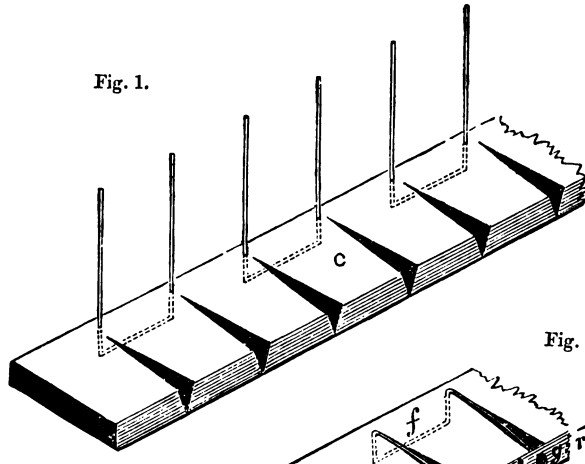
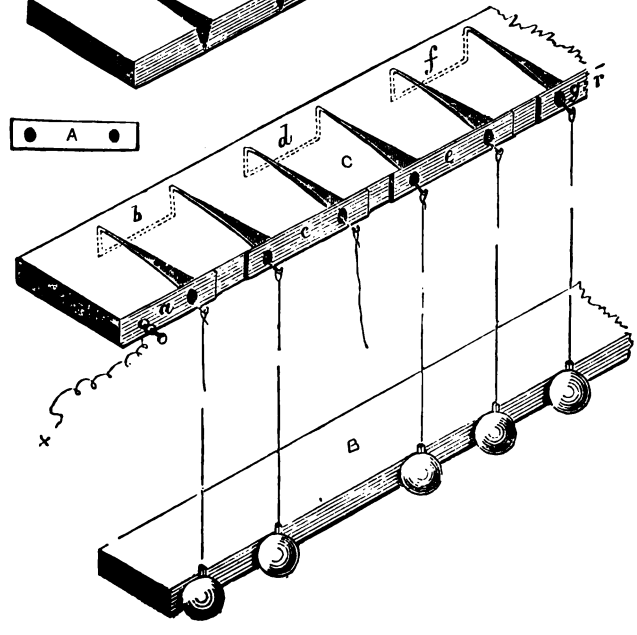


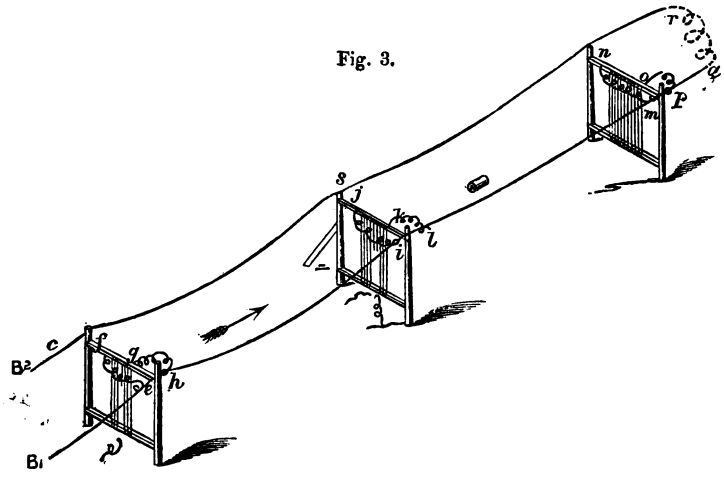
Fig. 2.



spring-wire (No. 14 or 15), are fixed with their prongs in the continuation of the grooves. Pieces of sheet copper, *A*, are provided, having two elliptical holes, the distance of whose centres equals the distance of the grooves. The pieces of copper, *A*, are used to connect each wire staple, as *C*, with its neighbour on each side. Thus (Fig. 2), *a*, *c*, *e*, *g*, &c., represent these copper connections put in their places and holding down the wire springs, which, when free, are in contact with the tops of the holes, but when *properly weighted* rest on the lower edge of the holes. The weights are heavy enough to keep the wire spring in contact with the lower edge of the holes. They are suspended by means of cotton threads. Thus the copper *c* forms a connection between the staples *b* and *d*, the copper *e* joins *d* and *f*, and so on. A galvanic stream will therefore take the following course, whether the springs be weighted or unweighted: copper *a*, brass *b*, copper *c*, brass *d*, copper *e*, brass *f*, copper *g*, &c. The current will only be interrupted when one or more threads have been cut and the corresponding spring is flying from the bottom to the top of its hole. About $\frac{1}{50}^{\text{th}}$ of a second is required for the complete registration of such an interruption, the spring traversing about half-an-inch. The shelf, *B*, is placed for the weights to rest against, partly to prevent them from being carried forward by the shot, but chiefly to prevent the untwisting of the threads which support the weights.

The arrangement of the screens for an experiment is shown in Fig. 3. The wires for conveying the galvanic current are, like the common telegraph wire, carried on posts. B_1, *h*, *l*, *p*, *q*, *r*, *n*, *s*, *c*, B_2, is a continuous piece of wire; but there are interruptions between *e* and *h*, between *i* and *l*, between *m* and *p*, &c., in order to make the

Fig. 3.

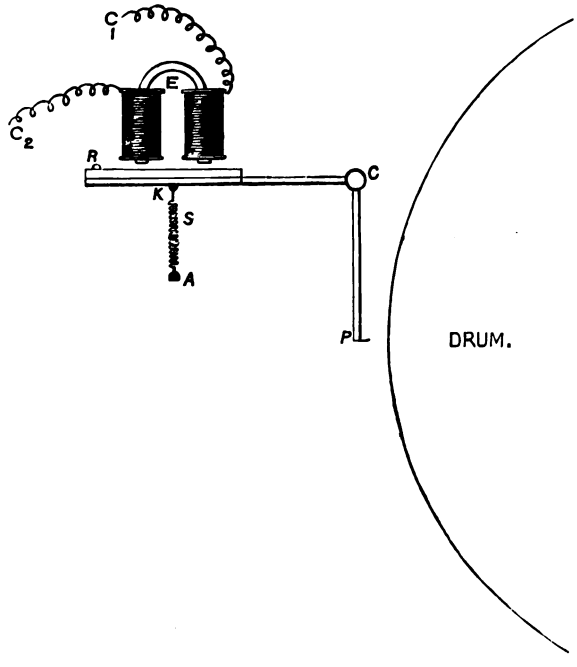


galvanic current circulate through the screens. The course of the galvanic current is B_1 *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *c*, B_2. The ends, B_1 B_2, are connected with the instrument and battery. The shot, being fired through the screens, in passing cuts one or more

threads at each screen; so that corresponding to the instant at which the shot passes each screen there is an interruption of the galvanic current, and a simultaneous record on the drum of the chronograph.

The chronograph consists of a smooth metal vertical drum attached to a fly-wheel. The drum is covered with blue glazed paper. There is a stage (carrying an electro-magnet and keeper) which can be raised or lowered at pleasure by means of a screw. The arrangement is represented in plan in Fig. 4. The stage is not shown, but the lever and electro-magnet are secured to it, and move with it. The

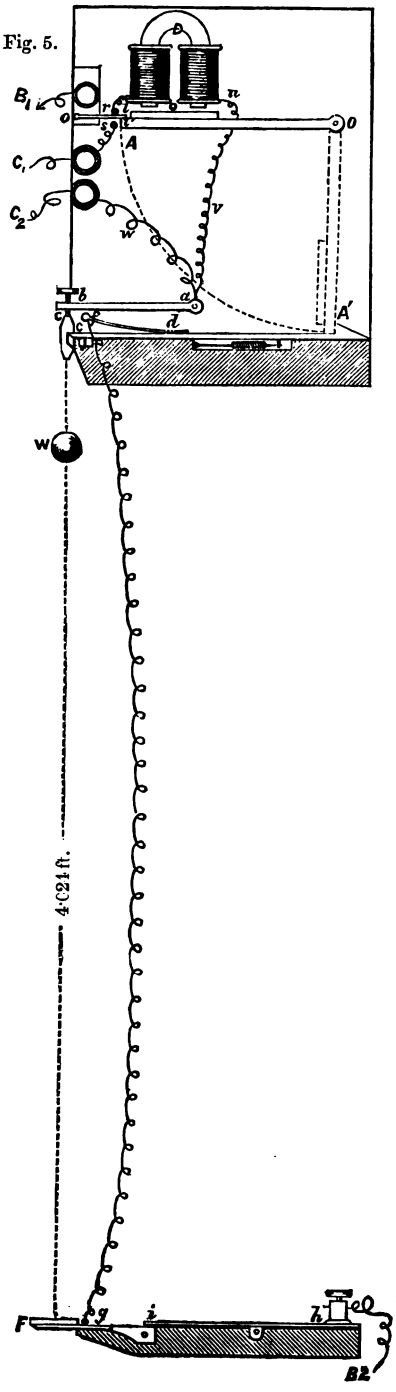
Fig. 4.



keeper, *K*, is in one piece with the bent lever, *KCP*, of which the fulcrum, *C*, is fixed. The other extremity of the lever carries a pricker, *P*, by means of which, when the current is broken, a record is made on the drum. This lever is held also by a spring, *S*, which can be made stronger or weaker as required. The strength of the spring must be so regulated that when the current passes through the electro-magnet, the keeper, *K*, should overcome the resistance of the spring and move towards the magnet. It is never allowed to come into actual contact with the poles of the magnet, but is stopped by a projection, *R* (so as to avoid the evil effects of remaining magnetism). When the current is broken at the screens, or at any other part of the circuit, the electro-magnet becomes demagnetised, and the spring, *S*, suddenly draws back the lever, and causes the pricker, *P*, to strike the drum, and to make a mark on the paper.

The time (half a second) is measured by a weight, *W*, falling freely through a space of 4·024 ft. The arrangement for dropping the ball, *W*, which gives the scale of time, is shown in Fig. 5. The wires, C_1C_2, must be connected with the electro-magnet, *E*, of the chronograph, shown in Fig. 4. The wire B_1 must be connected with the battery, thence to the wire B_1 in the screens (Fig. 3), and the wire B_2 (Fig. 5) must be connected with B_2 (Fig. 3). The current now circulates through the screens and the electro-magnet in the chronograph.

Fig. 5.



The firing of the gun is caused by the fall of another weight. When the weight is dropped, a momentary interruption of the current is caused.

Fig. 5 shows the arrangement by which the current is turned out of the chronograph when an experiment is about to be made. The lever, *OA*, is pressed up into the horizontal position by hand, the end of it, *A*, carrying up with it a brass spring or *switch* (which naturally rests on *s*) to the projection, *r*, in connection with the electro-magnet, *D*, which consequently becomes magnetised, and *holds* the lever, *OA*, in the horizontal position. The current is thus diverted from *s*, in connection with the chronograph, and the electro-magnet, *E*, is demagnetised so long as the lever, *OA*, remains in a horizontal position.

The instrument is represented in Fig. 5 ready prepared for the firing of a round, the electro-magnet, *E* (Fig. 4), of the chronograph being excluded from the circuit. When the weight intended to fire the gun is liberated, the momentary interruption of the current destroys the magnetism in the electro-magnet, *D*, and the lever, *OA*, begins to fall from its own weight. The *switch* descends from the posi-

tion *or* in contact with *r*, into the position *os* in contact with *s*, and so turns the current into the course B_1, *o*, *s*, C_1—through the electro-magnet, *E*, in the chronograph—C_2, *w*, *a*, *b*, *c*, *c*, *d*, *f*, *g*, *h*, B_2. When the lever, *OA*, comes into the vertical position, *OA'*, it strikes a projection on the bolt, *cA'*, and thus suddenly withdraws in a horizontal direction the support of the ball, *W*. Since the current passed through *c*, *c*, *d*, it is quite certain that the *interruption* of the current and the *dropping* of the ball are *simultaneous* events. The ball, *W*, is connected with *cc* by a flexible chain. The distance between the lower side of the ball, *A*, and the horizontal disc, *F*, is accurately adjusted at 4·024 ft.—being the distance in which a body falls freely in *half-a-second*. When the ball strikes the stage, *F*, the current is instantaneously broken at *g*.

When the ball, *W*, commences to fall the current is broken, and a mark is made on the drum of the chronograph; then the shot breaks the current when passing through the screens (usually three), thus giving three marks; and finally, when the ball strikes the stage, *F*, the current is again broken, and a fifth mark is registered. Thus,

$$A \qquad (1)_a \qquad (2)_b \qquad (3)_c \qquad B$$

represents the records on the cylinder, when the paper is unfolded and cut off. *AB* represents half-a-second, (1) to (2) the time (on the same scale) the shot takes to pass from the first to the second screen, (2) to (3) the time from second to third screen. The second screen is not necessary for the observation of a velocity, but is only introduced as a check to see that the records are consistent.

In calculating the velocity of the shot, the time from (1) to (3) screen, or time in *ac*, should be used. Now,

$$\text{time in } ac : \text{time in } AB :: ac : AB,$$

$$\text{or} \quad " \quad " \quad : \tfrac{1}{2} \text{ sec.} :: ac : AB,$$

$$\text{or} \quad " \quad " \quad = \tfrac{1}{2} \frac{ac}{AB} \text{ secs.};$$

and if *S* be the distance between the first and third screens, then velocity at the second screen—*i.e.*, half way between the first and third screens—is

$$\frac{S}{\text{time in } ac} = 2S \times \frac{AB}{ac} \text{ f.s.}$$

In an actual experiment at the Royal Laboratory rifle range, where the Gravity Chronograph is employed for the determination of the muzzle velocity of Small-arms, the distance $S = 200$ ft. *AB* is measured on a scale of equal parts, and *ac* on the same scale. For example: the velocity of the Martini-Henry rifle with the service charge is required. The fly-wheel of the chronograph is spun by hand, which rotates the drum on which the paper is pasted; the rifle is fired by means of a weight acting on the trigger, the release of which weight momentarily breaks the current, demagnetising the electro-magnet, *D*; the lever, *OA*, is released (the current circulating

in the chronograph), and when the lever reaches the vertical position cuts away the support of the ball, *W*. The current is momentarily broken, the electro-magnet, *E*, in the chronograph is demagnetised, and a mark, *A*, is made by the pricker; then the bullet passes through (1) (2) (3) screens in succession, momentarily breaking the current, and giving corresponding marks, *a*, *b*, *c*, on the drum; and finally the ball, *W*, strikes the horizontal disc, *F*, again breaking the current, and giving the mark, *B*, on the drum.

AB is then measured, and found $= 46{\cdot}06$,

ac " " " $= 14{\cdot}39$,

then velocity at second screen $= 2S \times \frac{AB}{ac} = 400 \times \frac{46{\cdot}06}{14{\cdot}39}$

$= 1280$ f.s.

The second screen being 150 ft. from the muzzle, the actual muzzle velocity must be calculated by means of the Distance and Velocity Table (p. 55), thus—

$$\frac{d^2}{w} S = S_v - S_V;$$

$$\text{or } S_V = S_v - \frac{d^2}{w} S = S_{1280} - \frac{(\cdot 45)^2}{\cdot 06857} \times 150,$$

$$S_v = 1976{\cdot}5$$
$$- \quad 442{\cdot}9$$
$$1533{\cdot}6$$

$$\therefore V = 1363 \text{ f.s.}$$

or muzzle velocity of this particular round with the Martini-Henry rifle is found to be 1363 f.s.

Watkin's Electric Chronograph.

Captain Watkin, R.A., has invented an Electric Chronograph, a full description of which is given in "Proceedings, R.A. Institution," Vol. IX. p. 14. The screens are on the same principle as Bashforth's, but the method of registration of the breaks in the current owing to the passage of the shot through the screens is different. This is done by means of a *spark* from the induced current of a Ruhmkorff coil. There are two vertical brass cylinders, blackened over for experiment. Between these cylinders a weight falls freely; this weight is released a short time before the gun is fired, and descends between the cylinders. The shot, on passing through the first screen, breaks the continuity of the primary wire of the coil, thus causing an induced spark to pass from one cylinder to the other through the brass wire of the weight—this being its shortest path. As the cylinders are covered with smoked paper, a minute spot registers the exact position of the weight at that moment. The weight continuing to fall, as the shot passes the second screen (the primary current being in the meantime re-established), the same result follows; and so on for any number of screens. The distance between the spots, as read off on the

velocity scale at the side of the cylinder, gives the time taken by the shot to pass the various screens. By means of a calculating scale the velocity may be at once ascertained for any distance between screens.

Experiments to test these three instruments were carried on at the Royal Laboratory rifle range in 1875, the instruments all registering the same shot, with the following results:—

Velocities of Small-Arm Bullets observed with the different Instruments.

Le Boulengé.		Bashforth.	Watkin.	
Instrument No. 78.	Instrument No. 79.		Right cylinder.	Left cylinder.
1167	1163	1163	1155	1157
1141	1140	1143	1137	1136
1140	1138	1138	1129	1132
1136	1133	1141	1138	1139
1147	1145	1142	1140	1140
Means of 5 rounds 1146	1144	1146	1140	1141
1272	1275	1269	1259	1259
1268	1264	1269	1267	1267
1270	1271	1277	1270	1268
1277	1279	1287	1272	1272
1285	1287	1290	1287	1285
1290	1294	1280	1284	1284
1264	1267	1263	1264	1264
1287	1289	1291	1285	1284
Means of 8 rounds 1277	1278	1275	1271	1273

The results show that there is no practical difference in the mean velocity, whichever instrument is employed.

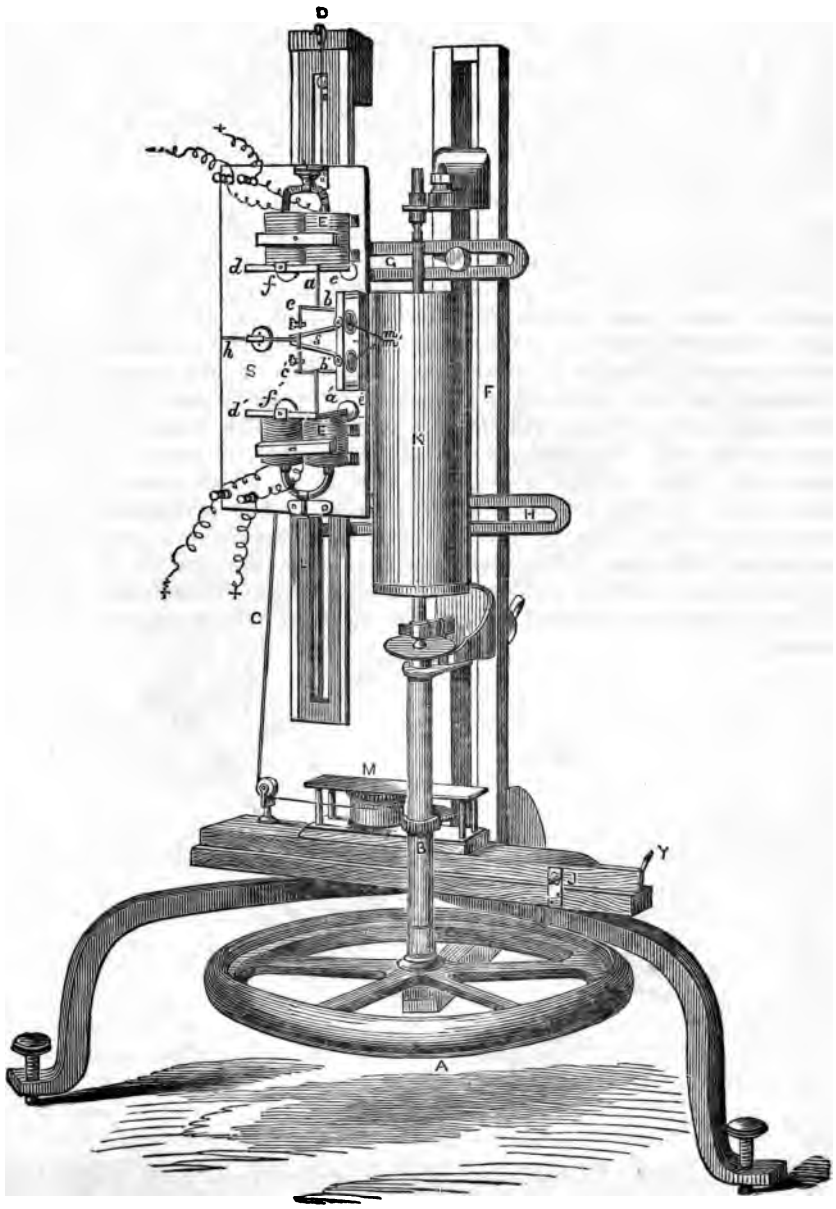
Bashforth's Clock Chronograph, for determining the resistance of the air.

It was by means of the Clock Chronograph, devised by Professor Bashforth himself, that he successfully carried out the series of experiments on the *resistance of the air*, already alluded to in this work.

The following is a description of the chronograph as constructed, and of various useful appendages:—Fig. 6 gives a general view of the chronograph. *A* is a fly-wheel capable of revolving about a vertical axis, and carrying with it the cylinder, *K*, which is covered with prepared paper for the reception of the clock and screen records. The length of the cylinder is 12 or 14 ins., and the diameter 4 ins. *B* is a toothed-wheel which gears with the wheelwork, *M*, so as to allow the string, *CD*, to be slowly unwrapped from its drum. The other end of *CD* being attached to the platform, *S*, allows it to descend slowly along the slide, *L*, about $\frac{1}{4}$ in. for each revolution of the cylinder. *E E'* are electro-magnets, *d d'* are frames supporting the keepers, and *f f'* are the ends of the springs which act against the attraction of the electro-magnets. When the current is interrupted in one circuit, as *E*, the magnetism of the electro-magnet is destroyed, the spring, *f*, carries back the keeper, which by means of the arm, *a*, gives a blow to the lever, *b*. Thus the marker, *m*, is made to depart from the uniform

spiral it was describing. When the current is restored the keeper is attracted, and thus the marker, *m*, is brought back, which continues to trace its spiral as if nothing had happened. *E'* is connected with the clock, and its marker, *m'*, records the seconds. *E* is connected with

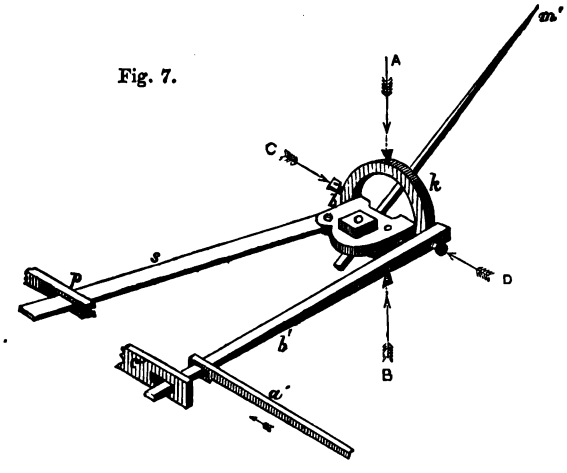
Fig. 6.



the screens, and records the passage of the shot through the screens. By comparing the marks made by *m m'* the exact velocity of the shot can be calculated at all points of its course. The slide, *L*, is fixed parallel to *F* and the cylinder, *K*, by the brackets, *GH*. *Y* is a screw for drawing back the wheelwork, *M*, and *J* a stop to regulate the distance between *M* and *B*. The depression of the lever, *h*, raises the springs, *s*, which act as levers, and bring the diamond points, *m m'*, down upon the paper. When an experiment is to be made, care is taken to see that the two currents are complete. The fly-wheel, *A*, is set in motion by hand, so as to make about three revolutions in two seconds. The markers, *m m'*, are brought down upon the paper, and after four or five beats of the clock the signal to fire is given; so that in about ten seconds the experiment is completed, and the instrument is ready for another.

Fig. 7 gives a full-sized view of one of the markers, showing the way in which it is moved. The depression of the lever, *h* (Fig. 6), raises *p*, and thus the lever, *s*, which is formed of watch-spring wire, brings down *m'* to the paper, and keeps it gently in contact. This motion takes place within the circle, *k*, about an axis, *CD*. *a'* is an arm connected with the electro-magnet. When the magnetism in *E'* is destroyed, *a'* begins to move away, and when it has moved a short distance it strikes the lever, *b'*, a sudden blow which carries it as far as the hole in the stop, *c'*, will allow it to move. The lever, *b'*, is rigidly connected with the circle, *k*, which is capable of moving about an axis, *AB*. This motion is communicated to *m'*, which describes a very short arc of a circle about a point in *AB*. The arrangement is so made that when either of the markers, *m m'*, is making a record, it has a motion which may be resolved partly in the direction of the motion of the paper under it, and partly in a direction perpendicular to this. Thus records are obtained which can be read off by scale with great nicety.

Fig. 7.

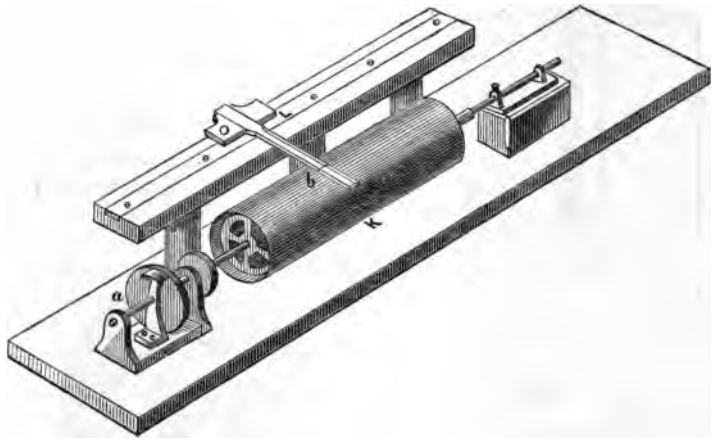


The pendulum of a half-seconds clock strikes once each double-beat

a very light spring, and so interrupts the galvanic current in *E'* once a second. The arrangement and formation of the screens is the same in principle as before described with the Gravity Chronograph. They are of course larger, to allow a margin of error in laying the gun, so that the shot may not cut away a portion of the framework of the screens. In the experiments for determining the resistance of the air 10 screens were used, each at 50 yds. interval. The shot, being fired through the screens, in passing cuts one or more threads at each screen, so that corresponding to the instant at which the shot passes each screen there is an interruption of the galvanic current, and a simultaneous record on the paper.

When the cylinder is filled with spirals—that is, after five or six shots—it is transferred to the instrument (Fig. 8), where *a* is a circle divided into 300 equal parts, and the division is carried to 3000 by the help of a vernier. A small T-square, having a fine edge at *b*, moves along a brass straight-edge, *L*, adjusted so as to be parallel to the axis of the cylinder. The mark *b* is carefully placed opposite each record on the paper by means of a tangent screw (not shown in the figure), and the vernier is read.

Fig. 8.



The Committee of Reference appointed by the War Office to report on this chronograph in comparison with other chronographs, state ("Reports on Experiments with the Bashforth Chronograph," p. 158) that "in Professor Bashforth's chronograph the time is directly measured by means of a clock with a half-seconds pendulum. Every double vibration of the pendulum is recorded on the revolving cylinder by the breaking of the same galvanic current under exactly the same conditions, and the time of the cannon ball's passing through each of the screens is recorded on the same revolving cylinder by the *momentary* interruption of a second galvanic current under exactly the same conditions.

"Thus the time scale required for converting the records made by this instrument into time intervals is here *supplied by the instrument itself*, and consequently the results obtained by it are unaffected by

any error of the same kind as those to which we have seen the former instruments to be liable. This freedom from constant errors and from errors of reduction is one of the great advantages of Professor Bashforth's instrument."

Instruments for determining the pressure in the gun and the velocity of the projectile inside the gun.

The instruments employed for determining the pressure of the powder in the bore of a gun are (*vide* p. 25)—

The Crusher Gauge,
The Noble Chronoscope.

The Crusher Gauge.

* The Crusher-Gauge is employed for registering the maximum pressure of the powder gas at the particular point in the bore where the apparatus is placed. It superseded the Rodman "Pressure-Gauge" employed by the Committee on Explosives in their earlier experiments, on account of the extreme variability of the results given by that instrument.

THE CRUSHER-GAUGE.

Fig. 9.

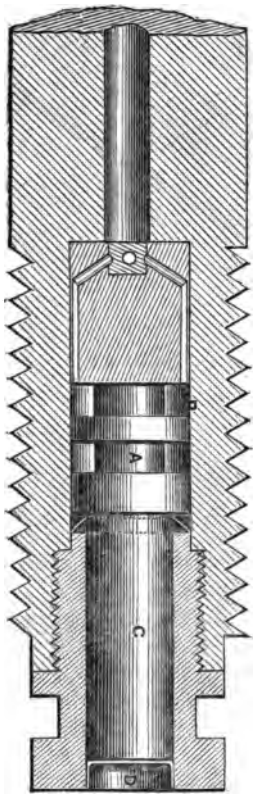
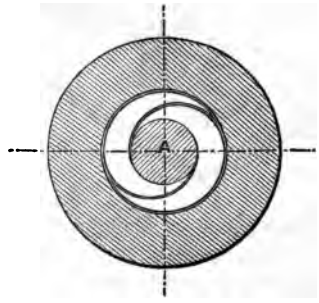


Fig. 10.



* Extracted from "Proceedings, R.A. Institution," Vol. IX. p. 368.

This exceedingly simple machine is shown above. The idea of ascertaining the pressure by the amount of "crushing" or compression of small cylinders of copper, originated in the Royal Gun Factories—in which department these gauges were first manufactured. The instrument consists of a steel bush (Fig. 9), of the same dimensions as the vent-bush used for all our service guns, provided with a moveable base or nozzle screwed on to the bush. The interior of the nozzle is hollowed out so as to form a chamber, in which is placed a steel "anvil," with grooves along its surface communicating with a channel corresponding with the vent-channel in a vent-bush. Thus any gas that may by accident find its way into the crusher-chamber has a free escape into the air. In this chamber is placed a small cylinder of copper, *A* (Figs. 9 and 10), half-an-inch in length and $\frac{1}{12}^{th}$ inch in diameter. This cylinder is held lightly (but not prevented from expanding laterally) in the centre of the chamber, by a small watch-spring. One end rests against the anvil, while the other end is acted on by a piston, *C* (Fig. 9), which is moveable in the nozzle, and is pressed against the copper cylinder by a small spring. The lower end of this piston is $\frac{1}{6}^{th}$ inch in area (under ordinary circumstances), and is open to the pressure of the powder gas; a small brass cup, or "gas-check," *D*, being inserted to prevent the gas from penetrating past the piston.

The apparatus is used as follows:—A hole is bored in the gun in the required position (see Plate V.), into which the gauge is screwed until the end of the plug is flush with the surface of the bore. On the explosion of the charge, the copper cylinder is crushed between the piston and the anvil, and the amount of compression is an indication of the pressure exerted by the gas at that point. The actual pressure corresponding to any given compression of the crusher-cylinder is arrived at by compressing a series of similar copper cylinders in the statical testing machine in the Royal Gun Factories, and tabulating the results. It is found that, with proper care in the selection and testing of the copper, the results are very reliable. Where extreme accuracy is required, each copper cylinder is tested previous to being inserted in the gauge. This is done by crushing them all up to a given pressure (say 18 tons on the square inch)—rather less than the pressure which the crusher-gauge is expected to record—which method of procedure has the additional advantage of eliminating some irregularities to which the crusher-gauge has been found, under certain circumstances, to be liable. The compression of the cylinder in the crusher-gauge is ascertained by removing the plug from the gun, unscrewing the nozzle, taking out the cylinder, and measuring its length by means of a micrometer reading to the one-thousandth of an inch. A reference to the tabulated results of the previous compression of similar coppers in the testing machine at once gives the pressure in tons on the square inch without any calculation. The gauge is cleaned, a fresh copper cylinder is inserted in the chamber, and the nozzle is screwed home, when the apparatus is ready for immediate use.

The Noble Chronoscope.

* The Noble Chronoscope is employed, as before stated, for measuring the time taken by the shot in passing over certain small successive intervals in the bore of the gun, and calculating therefrom the velocity of the shot in passing over each interval, and ultimately the mean pressure acting on it through that interval. It is the invention of Captain A. Noble, late R.A.—a member of the firm of Sir W. G. Armstrong & Co., in whose works at Elswick the instrument was made.

The following description is taken from the first Progress Report of the Committee on Explosives, dated February 1870, some slight corrections being made, so as to include the latest improvements in the instrument and in the method of employing it.

The principle of action consists in registering, by means of electric currents upon a recording surface travelling at a uniform and very high speed, the precise instant at which a shot passes certain defined points in the bore.

It consists of two portions—firstly, the *mechanical arrangement* for obtaining the necessary speed and keeping that speed uniform; secondly, the *electrical recording arrangement.*

The first part of the instrument consists of a series of thin metal discs, *AA* (Plate V., Fig. 1), each 36 ins. in circumference, fixed at intervals upon a horizontal shaft, *SS*, which is driven at a high speed by a heavy descending weight acting on a chain, *B*—arranged according to a plan originally proposed by Huyghens, through a train of gearing, *F*, multiplying 200 times. The driving weight is, during the experiment, continually wound up by means of the handle, *T*.

If the requisite speed of rotation were got up by the action of the falling weight alone, a considerable waste of time would ensue. To obviate this inconvenience, the required velocity can be obtained with great rapidity by means of the handle, *C*.

The precise rate of the discs is ascertained by means of the stop-clock, *D*, which can, at pleasure, be connected or disconnected with the revolving shaft, *E* (Fig. 1.), and the time of making any number of revolutions of this shaft can be recorded with accuracy to the one-tenth part of a second.

The speed usually attained in working the instrument is about 1100 ins. per second linear velocity at the circumference of the revolving discs, so that each inch travelled at that speed represents somewhat less than the one-thousandth part of a second; and as the inch is subdivided by the vernier, *V*, into a thousand parts, a linear representation, at the circumference, is thus obtained of intervals of time as minute as the one-millionth part of a second. As a small variation in speed would affect the relation between the several records obtained, the uniformity of rotation is ascertained on each occasion of experiment by three observations—one immediately before, one during, and one immediately after the experiment—the mean of the three observations being taken for the average speed.

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* Extracted from "Proceedings, R.A. Institution," Vol. IX. p. 371.

With a little practice, there is no difficulty in arranging the instrument so that the discs may rotate either uniformly, or at a rate very slowly increasing or decreasing.

The *modus operandi* is as follows:—When the requisite speed of rotation has been attained, the stop-clock is connected with the shaft, *E*, and the time occupied by the wheel, *F*, in making five revolutions—that is, the time occupied by the discs, *A*, in making 1000 revolutions—is recorded.

After the wheel, *F*, has made one revolution, unconnected with the clock, the time of making 1000 revolutions is again observed, and in the middle of this observation the gun is fired.

After the wheel, *F*, has again made one revolution, the time of 1000 revolutions is once more recorded, and the instrument is stopped.

An idea of the degree of uniformity realised, when the instrument is in working order, will be furnished by the observations of speed of the instrument for six consecutive rounds, which are here given:—

CLOCK READINGS.—EXPERIMENTS WITH 38-TON 12-IN. GUN.

Round No. 14.

	Observed time. secs.
1st observation, 1000 revolutions made in	32·0
2nd " "	31·9
3rd " "	31·8

Round No. 15.

1st observation, 1000 revolutions made in	32·0
2nd " "	31·9
3rd " "	31·9

Round No. 16.

1st observation, 1000 revolutions made in	31·5
2nd " "	31·7
3rd " "	31·6

Round No. 17.

1st observation, 1000 revolutions made in	31·5
2nd " "	31·4
3rd " "	31·5

Round No. 18.

1st observation, 1000 revolutions made in	31·6
2nd " "	31·6
3rd " "	31·6

Round No. 19.

1st observation,	1000 revolutions	made in	32·0
2nd	"	"	32·1
3rd	"	"	32·2

The maintenance of the speed with so great a degree of uniformity is obtained by means of very great accuracy of workmanship in all parts of the mechanism.

The arrangements for obtaining the *electrical records* are as follow:—The edges of the revolving discs are covered with smoke by means of an oil lamp, and are connected with one of the secondary wires, *G* (Fig. 1), of an induction coil. The other secondary wire, *H*, carefully insulated, is brought to a discharger, *Y*, opposite the edge of its corresponding disc, and is fixed so as to be just clear of the latter.

When a spark passes from the discharger to the disc a minute portion of the lamp-black is burnt away upon that part of the disc which was opposite to the discharge at the instant of the passage of the spark. A distinct spot is left on the blackened disc, the lamp-black at that point having been burnt away by the spark, so that the metal is shown beneath.

The mode of connecting the primary wires of the induction coils with the bore of the gun in such a manner that the shot, in passing a defined point, shall sever the primary current, and thereby produce a spark from the secondary, is shown in Plate VI., Figs. 1, 2, and 3, representing a longitudinal and a transverse section of the bore, *B*, of the gun, along which the shot, *A*, is moving.

A hollow plug, *C*, is screwed into the gun, carrying at the end next the bore a cutter, *D*, which projects slightly into the bore. The cutter is held in this position by the primary wire, *e*, which passes in at one side of the plug, *C*, then through a hole in the cutter, *D*, and out again at the other side of the plug. The two ends of this wire are connected with the main wires leading to the instrument when the plugs have been fixed in the gun.

When the shot, *A*, is fired, it presses the cutter into the second position, as shown in the figure, thereby severing the primary wire, and causing the induced spark instantaneously to pass from the discharger to the disc, its passage being marked by the spot left upon the edge, as already described. To prevent a possibility of the cutter being forced down by any gas that may escape past the projectile, a safety-pin, *f*, secures the cutter firmly in its place. This safety-pin is cut simultaneously with the primary wire.

The mode of reading the observations is as follows:—After an experiment has been made, and the series of sparks has been found on the discs, the mark on No. 1 disc is, by means of a micrometer-screw, brought precisely opposite to the point of the discharger; the vernier, *V*, is then attached to the extremity of the shaft, *S*, is firmly fixed there, and is set at zero.

The spark of No. 2 disc is then brought precisely opposite to its dis-

charger, the vernier-reading is recorded, and so with the other discs in succession.

It is obvious that the results furnished by an instrument intended to measure the very small intervals of time necessary for the investigations in which the Committee on Explosives have been engaged, would be liable to be received with great suspicion were there no means of testing the accuracy of the instrument; accordingly, one of the chief objects in its design was to arrange that the accuracy of the indications could at any time be tested with facility.

Each disc, discharger, and induction coil, form, so to speak, an independent instrument for recording the spark when the primary wire is broken; and it is obvious that, if the whole of the primaries are cut simultaneously, the sparks on all the discs should be in a straight line, and the deviations from a straight line—that is, from an absolutely simultaneous record—give the instrumental errors.

Great difficulties have, however, been experienced in securing a simultaneous rupture of the primary wires, and the method found most satisfactory has been to connect each circuit with a fine insulated wire, and to wind each of these fine wires tightly round the body of a detonator, containing about 100 grs. of fulminate of mercury. On firing the detonator by electricity, the whole of the wires may be cut nearly simultaneously, and the action of the instrument examined.

The following table shows the mean errors of several consecutive readings of the instrument, the primary wires having been severed in the manner described; and it will be observed how exceedingly slight they are. It is also more than probable that, small as these errors are, they must be in great part ascribed to the impossibility of securing an instantaneous rupture of the currents in all the wires.

MEAN ERRORS.—SIMULTANEOUS OBSERVATIONS.

1st Experiment.

April 21, 1875

	secs.		secs.
Disc 1 mean error	·000006	Disc 5 mean error	·000004
" 2 "	·000004	" 6 "	·000002
" 3 "	·000003	" 7 "	·000005
" 4 "	·000002	" 8 "	·000002

2nd Experiment.

April 21, 1875.

	secs.		secs.
Disc 1 mean error	·000004	Disc 5 mean error	·000003
" 2 "	·000006	" 6 "	·000005
" 3 "	·000005	" 7 "	·000003
" 4 "	·000007	" 8 "	·000000

April 22, 1875.

	secs.		secs.
Disc 1 mean error	·000004	Disc 5 mean error	·000003
" 2 "	·000004	" 6 "	·000006
" 3 "	·000005	" 7 "	·000006
" 4 "	·000005	" 8 "	·000000

April 28, 1875.

	secs.		secs.
Disc 1 mean error	·000008	Disc 5 mean error	·000002
" 2 "	·000006	" 6 "	·000003
" 3 "	·000004	" 7 "	·000011
" 4 "	·000002	" 8 "	·000003

The errors in none of these observations exceed a few *millionths of a second*, though, as before stated, they include any variation in the time of cutting the wires.

The gun shown in Fig. 3, Plate V., is an 18-ton 10-in. rifled M.L. gun, having a length of bore of 145·5 ins.

It was tapped at three places in the powder-chamber, to receive crusher-gauges. These holes could be closed with solid plugs when not required. The cutting plugs belonging to the chronoscope were eighteen in number, and were fitted into holes at various intervals along the bore, as shown in Plate V. When not required for the chronoscope these holes could be used for crusher-gauges, or were closed with solid plugs. The construction of the cutting plugs has been already described. The holes for the cutting plugs at the breech end of the bore were so arranged that the front edge of the projectile was flush with one of them with each of the different charges employed. The projectiles used in these and similar experiments are cast-iron cylinders, having a windage of only 0·01 in.

The loading of the gun is conducted as follows:—

The cartridge is made up to the required diameter; it is then inserted into the shot-chamber; the projectile is next pushed home gently until a stop on the rammer comes into contact with the face of the muzzle. Thus the head of the shot is brought to the same position in each round, and the cartridge always touches the base of the projectile.

The crushers and cutting plugs are then inserted into their respective holes and screwed home.

Finally, the wires of the cutting plugs are connected with the main wires leading to the instrument, and the tube is inserted in the vent.

After the discharge, the first operation is to wash out the gun; all the plugs are then removed and the holes dried preparatory to another round.

The coppers are taken from the crushers, stamped with the number of the round, and the measurements recorded.

The following table, extracted from the Report of the Committee on the 12·5-in. 38-ton gun, shows the velocity of the shot in passing over each foot of the bore from its initial position, and the pressures recorded

by the crusher-gauge. Charge, 130 lbs. 1·5-in. cubical powder; projectile, 800 lbs.

No. or letter of plug.	Mean of 4 rounds.			Mean of 3 rounds with gas-checks.		
	Pressure.	Velocity.	Difference in velocity.	Pressure.	Velocity.	Difference in velocity.
	tons per sq. in.	ft.	ft.	tons per sq. in.	ft.	ft.
A	23·8			23·1		
		137			125	
1	23·8		533	23·6		540
		670			665	
2	22·5		244	22·2		230
		914			895	
3	19·9		127	28·0		130
		1041			1025	
4	14·2		69	16·0		72
		1110			1097	
5	11·6		55	11·4		57
		1165			1154	
6	7·0		47	6·9		49
		1212			1203	
7	3·8		40	3·9		42
		1252			1245	
8	2·9		34	2·9		36
		1286			1281	
9			29			31
		1315			1312	
10			25	1·5		27
		1340			1339	
11			21	1·7		23
		1361			1362	
12	1·4		18	1·5		20
		1379			1382	
13	1·6		16	1·3		18
		1395			1400	
14	1·3		14	0·5		16
		1409			1416	
15	1·1					
16	0·6					

The object of the experiments from which the above rounds are taken was to ascertain the gain in velocity obtained by increasing the length of bore of the original 35-ton guns by 36 ins. If it be required to calculate the mean pressure at any point, it is necessary to measure the velocity of the projectile, during the first part of its motion in the bore, over much shorter intervals than a foot. Experiments of this nature were carried out in the 10-in. and 11-in. guns, the distance between the cutter plugs at the seat of the projectile being 0·2 ft. or 2·4 ins., as shown in Plate V., Fig. 3. The following table* gives the data obtained with the chronoscope for calculating velocity and pressure in

* "Researches on Explosives: Fired Gunpowder," by Captain Noble (late R.A.), F.R.S., &c., and F. A. Abel, F.R.S., President C.S., &c. From the "Philosophical Transactions of the Royal Society," Part I., 1875.

the bore of the 10-in. gun. Charge, 70 lbs. pebble powder; weight of projectile, 300 lbs.; muzzle velocity, 1527 ft. (*Vide* Plate I., p. 24.)

Distance from seat of shot.	Time observed at plugs.	Total time from seat of shot.	Time taken by shot to traverse distance between plugs.	Mean velocity over space between plugs.
ft.	secs.	secs.	secs.	ft.
0·00.....	.....	·000000		
			·002683	22
0·06.....	·000000	·002683		
			·001096	183
0·26.....	·001096	·003779		
			·000515	388
0·46.....	·001611	·004294		
			·000356	562
0·66.....	·001967	·004650		
			·000305	656
0·86.....	·002272	·004955		
			·000276	725
1·06.....	·002548	·005231		
			·000488	820
1·46.....	·003036	·005719		
			·000433	924
1·86.....	·003469	·006152		
			·000400	1000
2·26.....	·003869	·006552		
			·000375	1065
2·66.....	·004244	·006927		
			·000703	1138
3·46.....	·004947	·007630		
			·000658	1215
4·26.....	·005605	·008288		
			·000629	1273
5·06.....	·006234	·008917		
			·001192	1342
6·66	·007426	·010109		
			·001128	1418
8·26.....	·008554	·011237		

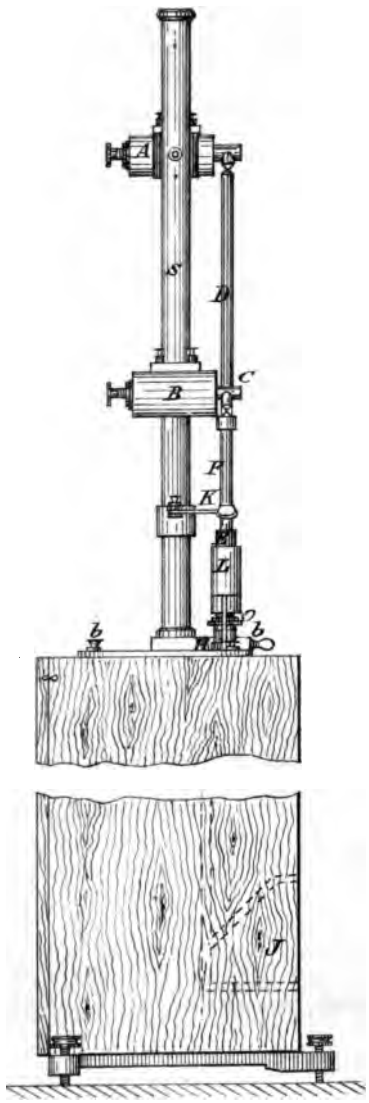
From these data are deduced, by correction and interpolation, the times occupied by the shot in passing over each two-hundredth ($\frac{1}{200}$) of a foot. From the differences of these times are calculated the mean velocities over each of these minute spaces, and, from the velocities, the mean pressures necessary to produce them are obtained. These calculations are extremely laborious. The rounds are worked out by Captain Noble (the inventor of the instrument) and an assistant, using two calculating machines, to check one another, and many hours of hard work are required in calculating the pressures for each round. The method of arriving at the velocities, as shown in the foregoing tables, is a comparatively simple matter.

PL. III.

Fig. 1. Chronograph in position for measuring the velocity of projectiles.

SCALE $\frac{1}{8}$ th "

Side view from the left.



Front view.

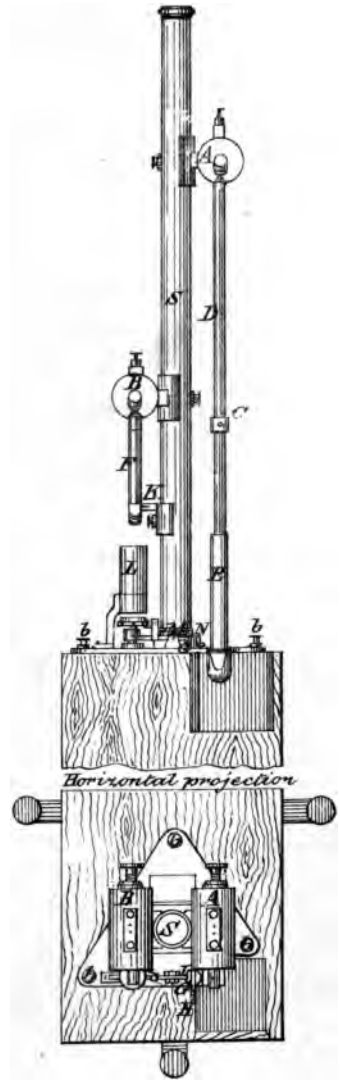
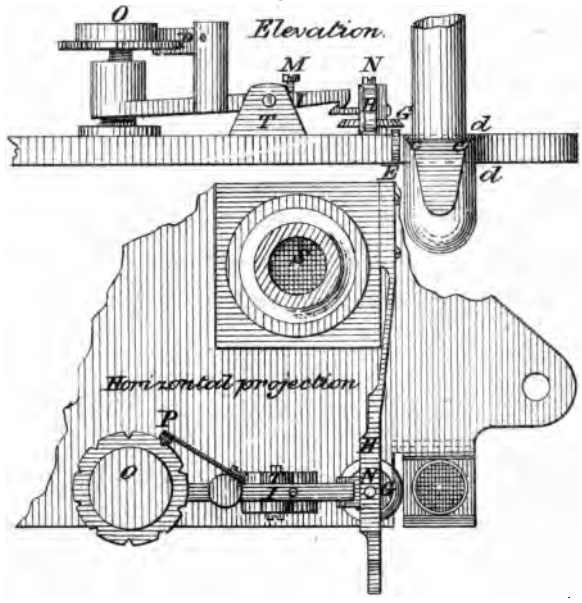


Fig 2. The Trigger. (Scale $\frac{1}{2}$.)



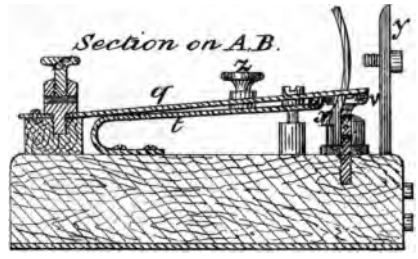
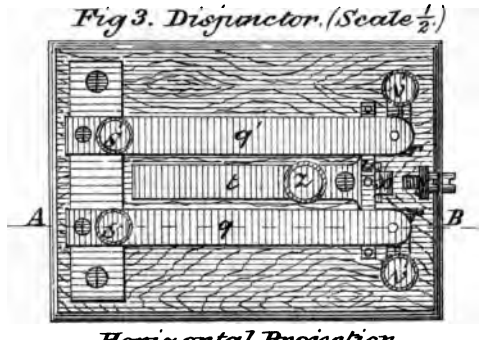
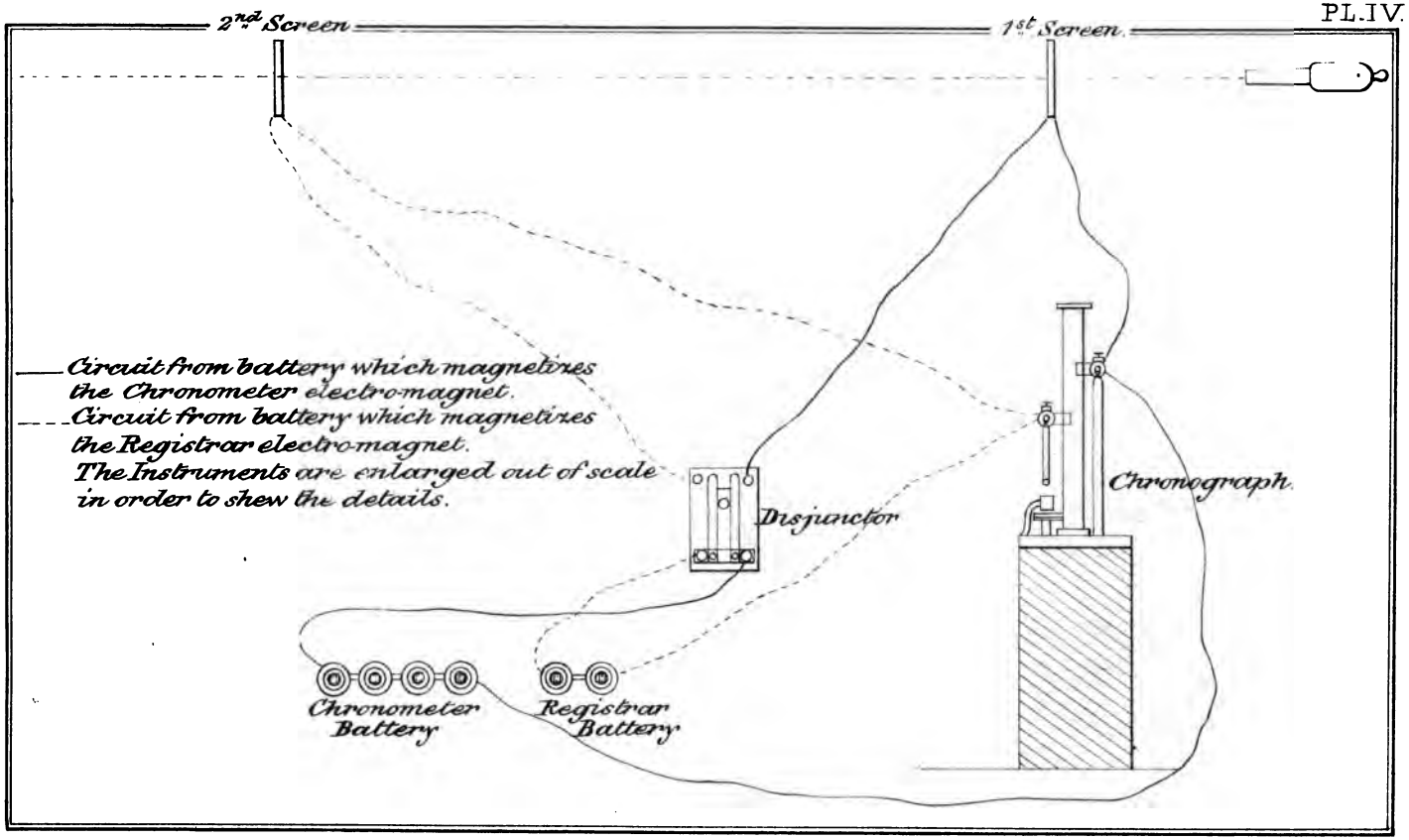


Fig 3. Disjunctor. (Scale $\frac{1}{2}$.)



Horizontal Projection.


2nd. Screen
1st. Screen.
Circuit from battery which magnetizes the Chronometer electro-magnet.
Circuit from battery which magnetizes the Registrar electro-magnet.
The Instruments are enlarged out of scale in order to shew the details.
Disjunctor
Chronograph.
Chronometer Battery
Registrar Battery

SCALE. INSTRUMENT ABOUT $\frac{1}{12}$TH
GUN ABOUT $\frac{1}{36}$TH

FIG 1.

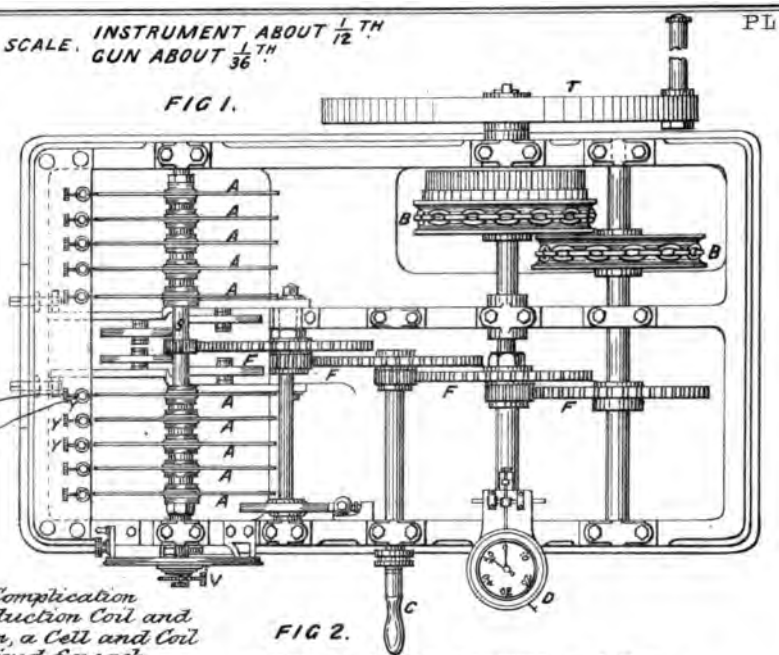


FIG 2.

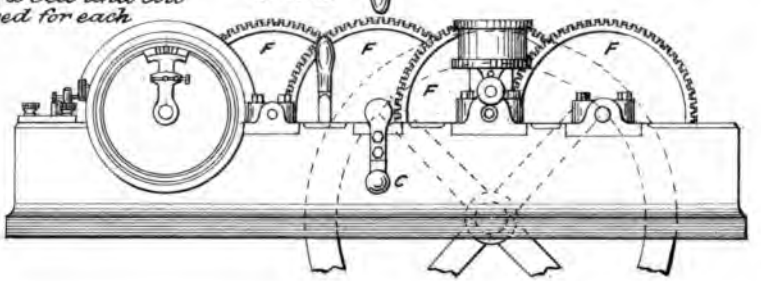
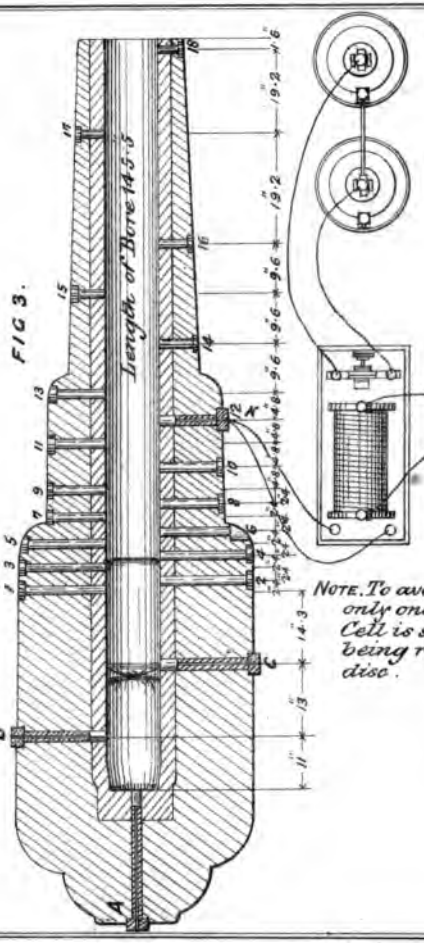
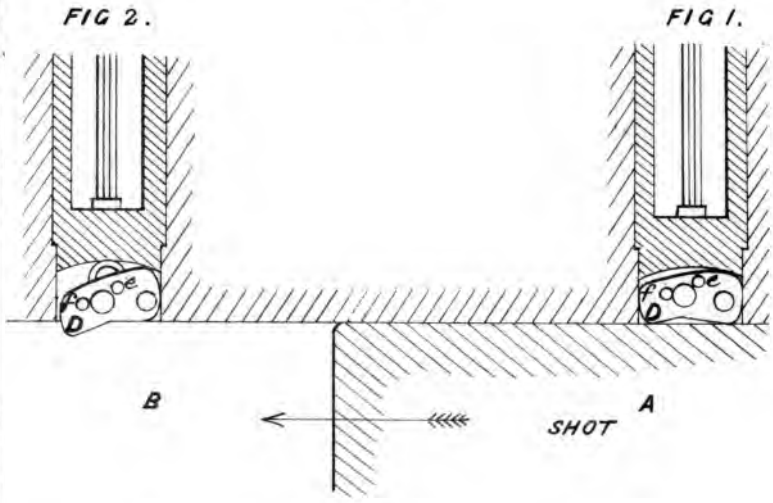
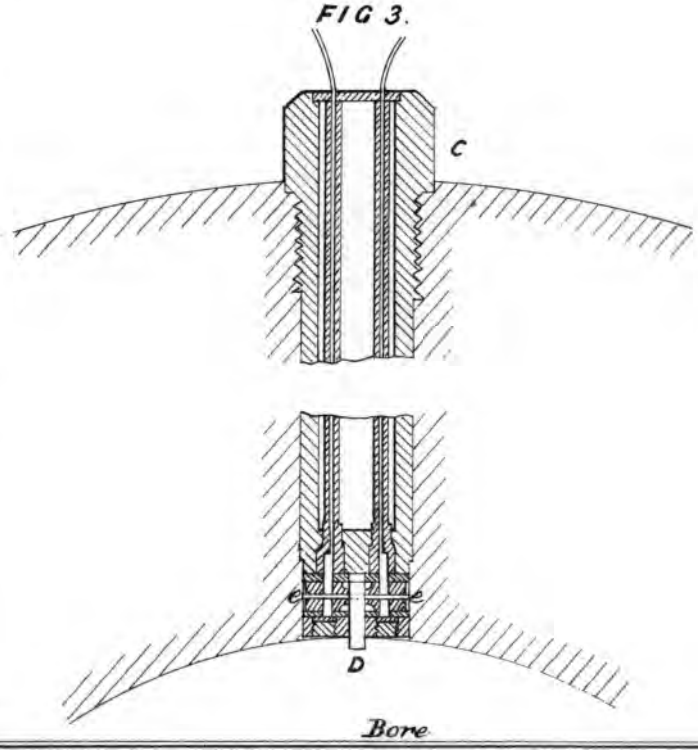


FIG 3.



NOTE. To avoid Complication only one Induction Coil and Cell is shewn, a Cell and Coil being required for each disc.

CUTTING PLUGS.

SCALE $\frac{1}{2}$.